OF THE LAWS of CHANCE, OR, A METHOD Of Calculation of the Hazards of GAME,

Plainly demonſtrated,

And applied to GAMES at preſent moſt in Uſe;

Which may be eaſily extended to the moſt intricate Caſes of CHANCE imaginable.

The FOURTH EDITION,
Revis'd by *JOHN HAM.*

By whom is added,

A Demonſtration of the Gain of the Banker in any Circumſtance of the Game call'd PHARAON;

And how to determine the Odds at the ACE of HEARTS or FAIR CHANCE;

With the Arithmetical Solution of ſome Queſtions relating to LOTTERIES;

And a few Remarks upon HAZARD and BACKGAMMON.

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PREFACE.

IT is thought as necessary to write a Preface before a Book, as it is judg'd civil, when you invite a Friend to Dinner, to proffer him a Glass of Hock beforehand for a Whet: And this being maim'd enough for want of a Dedication, I am resolv'd it shall not want an Epistle to the Reader too. I shall not take upon me to determine, whether it is lawful to play at Dice or not, leaving that to be disputed betwixt the Fanatick Parsons *and the* Sharpers; *I am sure it is lawful to deal with Dice as with other Epidemic Distempers; and I am confident that the writing a Book about it, will contribute as little towards its Encouragement, as Fluxing and Precipitates do to Whoring.*

It will be to little purpose to tell my Reader, of how great Antiquity the playing at Dice is. I will only let him know, that by the Aleæ Ludus, *the Antients comprehended all Games, which were subjected to the determination of mere Chance; this sort of Gaming was strictly forbid by the Emperor* Justinian, Cod. Lib. 3. Tit. 43. *under very severe Penalties; and* Phocius Nomocan. Tit. 9. Cap. 27. *acquaints us, that the Use of this was altogether denied the Clergy of that time.* Seneca *says very well,* Aleator quantò in arte est melior, tantò est nequior; *That by how much the one is more skilful in Games, by so much he is the more culpable; or we may say of this, as an ingenious Man says of Dancing, That to be extraordinary good at it, is to be excellant in a Fault; therefore I hope no body will imagine I had so mean a Design in this, as to teach the Art of Playing at Dice.*

A great part of this Discourse is a Translation from Mons. Huygen's *Treatise,* De ratiociniis in ludo Aleæ; *one, who in his Improvements of Philosophy, has but one Superior, and I think few or no Equals. The whole I undertook for my own Divertisement, next to the Satisfaction of some*

ſome Friends, who would now and then be wrangling about the Proportions of Hazards in ſome Caſes that are here decided. All it requir'd was a few ſpare Hours, and but little Work for the Brain; my Deſign in publiſhing it, was to make it of more general Uſe, and perhaps perſuade a raw Squire, by it, to keep his Money in his Pocket; and if, upon this account, I ſhould incur the Clamours of the Sharpers, I do not much regard it, ſince they are a ſort of People the World is not bound to provide for.

You will find here a very plain and eaſy Method of the Calculation of the Hazards of Game, which a man may underſtand, without knowing the Quadratures of Curves, *the Doctrine of* Series's, *or the Laws of* Concentripetation *of Bodies, or the Periods of the* Satellites *of* Jupiter; *yea, without ſo much as the Elements of* Euclid. *There is nothing required for the comprehending the whole, but common Senſe and practical Arithmetick; ſaving a few Touches of* Algebra, *as in the firſt Three Propoſitions, where the Reader, without ſuſpicion of Popery, may make uſe of a ſtrong implicit Faith; tho' I muſt confeſs, it does not much recommend it ſelf to me in theſe Purpoſes; for I had rather he*

would

would enquire, and I believe he will find the Speculation not unpleasant.

Every man's Success in any Affair is proportional to his Conduct and Fortune. Fortune (in the sense of most People) signifies an Event which depends on Chance, agreeing with my Wish; and Misfortune signifies such an one, whose immediate Causes I don't know, and consequently can neither foretel nor produce it (for it is no Heresy to believe, that Providence suffers ordinary matters to run in the Channel of second Causes). Now I suppose, that all a wise Man can do in such a Case is, to lay his Business on such Events, as have the most powerful second Causes, and this is true both in the great Events of the World, and in ordinary Games. It is impossible for a Die, with such determin'd force and direction, not to fall on such a determin'd side, only I don't know the force and direction which makes it fall on such a determin'd side, and therefore I call that Chance, which is nothing but want of Art; that only which is left to me, is to wager where there are the greatest number of Chances, and consequently the greatest probability to gain; and the whole Art of Gaming, where there is any thing of Hazard, will be

be reduc'd to this at last, viz. *in dubious Cases to calculate on which side there are most Chances; and tho' this can't be done in the midst of Game precisely to an Unit, yet a Man who knows the Principles, may make such a conjecture, as will be a sufficient direction to him; and tho' it is possible, if there are any Chances against him at all, that he may lose, yet when he chuseth the safest side, he may part with his Money with more content (if there can be any at all) in such a Case.*

I will not debate, whether one may engage another in a disadvantageous Wager. Games may be suppos'd to be a tryal of Wit as well as Fortune, and every Man, when he enters the Lists with another, unless out of Complaisance, takes it for granted, his Fortune and Judgment, are, at least, equal to those of his Play-Fellow; but this I am sure of, that false Dice, Tricks of Leger-de-main, &c. *are inexcusable, for the question in Gaming is not, Who is the best Jugler?*

The Reader may here observe the Force of Numbers, which can be successfully applied, even to those things, which one would imagine are subject to no Rules. There are very few things which we know, which

which are not capable of being reduc'd to a Mathematical Reasoning; and when they cannot, it's a sign our Knowledge of them is very small and confus'd; and where a mathematical reasoning can be had, it's as great folly to make use of any other, as to grope for a thing in the dark, when you have a Candle standing by you. I believe the Calculation of the Quantity of Probability might be improved to a very useful and pleasant Speculation, and applied to a great many Events which are accidental, besides those of Games; only these Cases would be infinitely more confus'd, as depending on Chances which the most part of Men are ignorant of; and as I have hinted already, all the Politicks in the World, are nothing else but a kind of Analysis of the Quantity of Probability in casual Events, and a good Politician signifies no more, but one who is dextrous at such Calculations; only the Principles which are made use of in the Solution of such Problems, can't be studied in a Closet, but acquir'd by the Observation of Mankind.

There is likewise a Calculation of the Quantity of Probability founded on Experience, to be made use of in Wagers about any thing; it is odds, if a Woman is

with

with Child, *but it ſhall be a* Boy; *and if you would know the juſt odds, you muſt conſider the Proportion in the Bills that the Males bear to the Females: The Yearly Bills of Mortality are obſerv'd to bear ſuch Proportion to the live People as* 1 *to* 30, *or* 26; *therefore it is an even Wager, that one out of thirteen, dies within a Year (which may be a good reaſon, tho' not the true, of that fooliſh piece of Superſtition), becauſe, at this rate, if* 1 *out of* 26 *dies, you are no loſer. It is but* 1 *to* 18 *if you meet a* Parſon *in the Street, that he proves to be a* Non-Juror, *becauſe there is but* 1 *of* 36 *that are ſuch. It is hardly* 1 *to* 10, *that a* Woman *of Twenty Years old has her* Maidenhead, *and almoſt the ſame Wager, that a* Town-Spark *of that Age has not been* clap'd. *I think a Man might venture ſome odds, that* 100 *of the* Gens d'arms *beats an equal Number of* Dutch Troopers; *and that an* Engliſh Regiment *ſtands its ground as long as another, making Experience our Guide in all theſe Caſes and others of the like nature.*

But there are no caſual Events, which are ſo eaſily ſubjected to Numbers, as thoſe of Games; and I believe, there the Speculation might be improved ſo far, as to bring in the Doctrine of the Series's *and* Logarithms.

richms. *Since Gaming is become a Trade, I think it fit the Adventurers ſhould be upon the Square; and therefore in the Contrivance of Games there ought to be a ſtrict Calculation made uſe of, that they mayn't put one Party in more probability to gain than another; and likewiſe, if a Man has a conſiderable Venture; he ought to be allow'd to withdraw his Money when he pleaſes, paying according to the Circumſtances he is then in: and it were eaſy in moſt Games to make Tables, by Inſpection of which, a Man might know what he was either to pay or receive, in any Circumſtances you can imagin, it being convenient to ſave a part of one's Money, rather than venture the loſs of it all.*

I ſhall add no more, but that a Mathematician will eaſily perceive, it is not put in ſuch a Dreſs as to be taken notice of by him, there being abundance of Words ſpent to make the more ordinary ſort of People underſtand it.

FOR the ſake of thoſe who are not verſ'd in *Mathematicks*, I have added the following Explanation of *Signs*.

$=$ *Equal.*

$+$ *More*, or to be added.

$-$ *Leſs*, or to be ſubtracted.

$\times$ *Multiplied.*

$\div$ *Divided.*

EXAMPLE.

$3 \times 4 + 3 - 1 = 14 = \frac{5}{9}a$, is to be read thus;

3 multiplied in 4, more by 3, leſs by 1, is equal to 14, which is equal to five ninth parts of *a*.

An Exact
METHOD
For SOLVING the
Hazards of GAME.

ALTHO' the Events of Games, which Fortune ſolely governs, are uncertain, yet it may be certainly determin'd, how much one is more ready to loſe than gain. For *Example:* If one ſhould wager, at the firſt throw with one Die, to throw ſix, it's an accident if he gains or not; but by how much it's more probable he will loſe than gain, is really determin'd by the Nature of the thing, and capable of a ſtrict Calculation. So likewiſe if I ſhould play with another on this condition, that the Victory ſhould be to the three firſt Games, and I had gain'd one al-

ready, it is ſtill uncertain who ſhall gain the third; yet by a demonſtrative Reaſoning, I can eſtimate both the Value of his Expectation and mine, and conſequently (if we agree to leave the Game imperfect) determine how great a ſhare of the Stakes belong to me, and how much to my Play-fellow; or if any were deſirous to take my place, at what rate I ought to ſell it. Hence may ariſe innumerable Queries among two, three, or more Gameſters: and ſince the Calculation of theſe things is a little out of the common Road, and can be oft-times apply'd to good purpoſe, I ſhall briefly here ſhew how it is to be done, and afterwards explain thoſe things which belong properly to the Dice.

In both Caſes I ſhall make uſe of this Principle, *One's Hazard or Expectation to gain any thing, is worth ſo much, as, if he had it, he could purchaſe the like Hazard or Expectation again in a juſt and equal Game.*

For *Example*, if one, without my Knowledge, ſhould hide in one hand 7 Shillings, and in his other 3 Shillings, and put it to my choice which Hand I would take, I ſay this is as much worth to me, as if he ſhould give me 5 Shillings; becauſe, if I have 5 Shillings, I can purchaſe as good a Chance again, and that in a fair and juſt Game.

PRO-

PROPOSITION I.

If I expect a *or* b, *either of which, with equal probability, may fall to me, then my Expectation is worth* $\frac{a+b}{2}$, *that is, the half Sum of* a *and* b.

THAT I may not only demonſtrate, but likewiſe inveſtigate this Rule, ſuppoſe the Value of my Expectation be x; by the former Principle having x, I can purchaſe as good an Expectation again in a fair and juſt Game. Suppoſe then I play with another on theſe terms, That every one ſtakes x, and the Gainer give to the Loſer a, this Game is juſt, and it appears, that at this rate, I have an equal hazard either to get a if I loſe the Game, or $2x-a$ if I gain; for in this caſe I get $2x$, which are the Stakes, out of which I muſt pay the other a; but if $2x-a$ were worth b, then I have an equal hazard to get a or b; therefore making $2x-a=b$, $x=\frac{a+b}{2}$, which is the Value of my Expectation. The Demonſtration is eaſy; for having $\frac{a+b}{2}$, I can play with another who will ſtake $\frac{a+b}{2}$ againſt it, on this condition, that the Gainer ſhould give to the Loſer a;

by this means I have an equal Expectation to get a if I loſe, or b if I win; for in the laſt caſe I get $a + b$ the Stakes, out of which I muſt pay a to my Play-fellow.

In *Numbers*: if I had an equal hazard to get 3 or 7, then by this Propoſition, my Expectation is worth 5, and it is certain, having 5, I may have the ſame Chance; for if I play with another, ſo that every one ſtakes 5, and the Gainer pay to the Loſer 3, this is a fair way of gaming; and it is evident I have an equal hazard to get 3 if I loſe, or 7 if I gain.

Prop. II.

If I expect a, b, *or* c, *either of which, with equal facility, may happen, then the Value of my Expectation is* $\frac{a+b+c}{3}$, *or the third part of the Sum of* a, b, *and* c.

FOR the Inveſtigation of which, ſuppoſe x be the value of my Expectation; then x muſt be ſuch, as I can purchaſe with it the ſame Expectation in a juſt Game: Suppoſe the Conditions of the Game be, that playing with two others, each of us ſtakes x, and I bargain with one of the Gameſters, if I win, to give him b, and he ſhall do the ſame to me; but with the

the other, that if I gain, I ſhall give him c, and *vice verſâ*; this is fair play: And here I have an equal hazard to get b, if the firſt win, c if the ſecond, or $3x - b - c$ if I gain myſelf; for then I get $3x$, *viz.* the Stakes, of which I give the one b and the other c; but if $3x - b - c$ be equal to a, I have an equal Expectation of a, b, or c; therefore making $3x - b - c = a$, $x = \frac{a+b+c}{3}$, which is the Value of my Expectation. After the ſame Method you will find, if I had an equal hazard to get a, b, c, or d, the Value of my Expectation $\frac{a+b+c+d}{4}$, that is the fourth part of the Sum of a, b, c, and d, &c.

PROP.

Prop. III.

If the number of Chances, by which a *falls to me, be* p, *and the number of Chances, by which* b *falls, be* q, *and supposing all the Chances do happen with equal facility, then the Value of my Expectation is* $\frac{pa+bq}{p+q}$; i. e. *the Product of* a *multiplied in the number of its Chances added to the Product of* b, *multiplied into the number of its Chances, and the Sum divided by the number of Chances both of* a *and* b.

SUppose, as before, x be the Value of my Expectation; then if I have x, I must be able to purchase with it that same Expectation again in a fair Game: For this I shall take as many Play-fellows as, with me, make up the number of $p+q$, of which let every one stake x, so the whole Stake will be $px+qx$, and every one plays with equal hopes of winning; with as many of my Fellow-Gamesters as the Number q stands for, I make this bargain one by one, that whoever of them gains shall give me b, and if I win, I shall do so to them; with every one of the rest of the Gamesters, whose Number is $p-1$, I make this bargain, that whoever of them gains,

gains, ſhall give me a, and I ſhall give every one of them as much, if I gain: It's evident this is fair play; for no Man here is injur'd; and in this caſe I have q Expectations to gain b, and $p - 1$ Expectations to gain a, and 1 Expectation (*viz.* when I win myſelf) to get $px + qx - bq - ap + a$; for then I am to deliver b to every one of the q Players, and a to every one of the $p - 1$ Gameſters, which makes $qb + pa - a$; if therefore $qx + bx - bq - ap + a$ were equal to a, I would have p Expectations of a (ſince juſt now I had $p - 1$ Expectations of it) and q Expectations of b, and ſo would have juſt come to my firſt Expectation; therefore putting $px + qx - bq - ap + a = a$, then is $x = \frac{ap + bq}{p + q}$.

In *Numbers;* If I had 3 Chances to gain for 13, and 2 for 8, by this Rule, my hazard is worth 11; for 13 multiplied by 3 gives 39, and 8 by 2 16, theſe two added, make 55, divided by 5 is 11; and I can eaſily ſhew, if I have 11, I can come to the like Expectation again; for playing with four others, and every one of us ſtaking 11, with two of them I make this bargain, that whoever gains ſhall give me 8, and I ſhall too do ſo to them; with the other two I make this bargain, that who-

ever

ever gains ſhall give me 13, and I them as much if I gain : it appears, by this means I have two Expectations to get 8, *viz.* if any of the firſt two gain, and 3 Expectations to get 13, *viz.* if either I or any of the other two gain; for in this caſe I gain the Stakes, which are 55, out of which I am oblig'd to give the firſt two 8, and the other two 13, and ſo there remains 13 for myſelf.

Prop. IV.

That I may come to the Queſtion propos'd, viz. *The making a juſt Diſtribution amongſt Gameſters, when their Hazards are unequal; we muſt begin with the moſt eaſy Caſes.*

SUppoſe then I play with another, on condition that he who wins the three firſt Games ſhall have the Stakes, and that I have already gain'd two, I would know, if we agree to break off the Game, and part the Stakes juſtly, how much falls to my ſhare?

The firſt thing we muſt conſider in ſuch Queſtions is the number of Games that are wanting to both: For *Example*, if it had been agreed betwixt us, that he ſhould have the Stakes who gain'd the firſt 20 Games,

Games, and if I had gain'd already 19, and my Fellow-Gamester but 18, my hazard is as much better than his in that case, as in this proposed, *viz.* When of 3 Games I have 2, and he but one, because in both cases there's 2 wanting to him, and 1 to me.

In the next place, to find the portion of the Stakes due to each of us, we must consider what would happen if the Game went on; it is certain, if I gain the first Game, I get the Stake, which I call a; but if he gain'd, both our Lots would be equal, and so there would fall to each of us $\frac{1}{2}a$; but since I have an equal hazard to gain or lose the first Game, I have an equal Expectation to gain a, or $\frac{1}{2}a$, which, by the first *Proposition*, is as much worth as the half Sum of both, *i. e.* $\frac{3}{4}a$, so there is left to my Fellow-Gamester $\frac{1}{4}a$; from whence it follows, that he who would buy my Game, ought to pay me for it $\frac{3}{4}a$; and therefore, he who undertakes to gain one Game before another gains two, may wager 3 to 1.

PROP. V.

Suppose I want but one Game, and my Fellow-Gamester three, it is required to make a just Distribution of the Stake.

LET us here likewise consider in what state we should be, if I or he gain'd the first Game; if I gain, I have the Stake a, if he, then he wants yet 2 Games, and I but 1, and therefore we should be in the same Condition which is supposed in the former Proposition; and so there would fall to my Share, as was demonstrated there, $\frac{3}{4}a$; therefore with equal facility there may happen to me a, or $\frac{3}{4}a$, which, by the first Proposition, is worth $\frac{7}{8}a$, and to my Fellow-Gamester there is left $\frac{1}{8}a$, and therefore my hazard to his is as 7 to 1.

As the Calculation of the former Proposition was requisite for this, so this will serve for the following. If I should suppose myself to want but one Game, and my Fellow four, (by the same Method) you will find $\frac{15}{16}$ of the Stake belongs to me, and $\frac{1}{16}$ to him.

PROP.

PROP. VI.

Suppose I want two Games, and my Fellow-Gamester three.

THEN by the next Game it will happen that I want but one, and he three, which (by the preceding Proposition) is worth $\frac{7}{8}a$; or that we should both want two, whence there will be $\frac{1}{2}a$ due to each of us: now I being in an equal probability to gain or lose the next Game, I have an equal hazard to gain $\frac{7}{8}a$ or $\frac{1}{2}a$, which by the first Proposition is worth $\frac{11}{16}a$; and so there are eleven parts of the Stakes due to me, and five to my Fellow.

PROP. VII.

Let us suppose I want two Games, and my Fellow four.

IF I gain the next Game, then I shall want but one, and my Fellow four; but if I lose it, then I shall want two, and he three: So I have an equal hazard for gaining $\frac{15}{16}a$, or $\frac{11}{16}a$, which, by the first, is worth $\frac{13}{16}a$: So it appears, that he who is to gain two Games for the other's four, is in a better condition than he who is to

gain one for the other's two; for my ſhare in the firſt caſe is $\frac{3}{4}a$ or $\frac{12}{16}a$, which is leſs than $\frac{13}{16}$, my ſhare in the laſt.

PROP. VIII.

Let us ſuppoſe three Gameſters, whereof the firſt and ſecond want 1 *Game, but the third* 2.

TO find the ſhare of the firſt, we muſt conſider what would happen if either he, or any of the other two gain'd the firſt Game; if he gains, then he has the Stake a; if the ſecond gain, he has nothing; but if the third gain, then each of them would want a Game, and ſo $\frac{1}{3}a$ would be due to every one of them. Thus the firſt Gameſter has one Expectation to gain a, one to gain nothing, and one for $\frac{1}{3}a$, (ſince all are in equal probability to gain the firſt Game) which by the ſecond Propoſition is worth $\frac{4}{9}a$: Now ſince the ſecond Gameſter's Condition is as good, his Share is likewiſe $\frac{4}{9}a$, and ſo there remains to the third $\frac{1}{9}a$, whoſe Share might have been as eaſily found by itſelf.

PROP.

PROP. IX.

In any number of Gamesters you please, amongst whom there are some who want more, some fewer Games: To find what is any one's share in the Stake, we must consider what would be due to him, whose Share we investigate, if either he, or any of his Fellow-Gamesters should gain the next following Game; add all their Shares together, and divide the Sum by the number of the Gamesters, the Quotient is his Share you were seeking.

SUppose three Gamesters *A*, *B*, and *C*; *A* wants 1 Game, *B* 2, and *C* likewise 2, I would find what is the share of the Stake due to *B*, which I shall call q.

First, we must consider what would fall to *B*'s Share, if either he, *A*, or *C*, wins the next Game; if *A* wins, the Game is ended, so he gets nothing; if *B* himself gain, then he wants 1 Game, *A* 1, and *C* 2; therefore, by the former Proposition, there is due to him in that case $\frac{4}{9}q$, then if *C* gains the next play, then *A* and *C* would want but 1, and *B* 2; and therefore, by the eighth Proposition, his Share would be worth $\frac{1}{9}q$; add together what is due to *B* in all these three Cases, *viz.* $0\frac{4}{9}q$, $\frac{1}{9}q$,

$\frac{1}{9}q$, the Sum is $\frac{5}{9}q$, which being divided by 3, the number of Gamesters, gives $\frac{5}{27}q$, which is the Share of *B* sought for: The Demonstration of this is clear from the second Proposition, because *B* has an equal hazard to gain $0\frac{4}{9}q$ or $\frac{1}{9}q$, that is $\frac{0+\frac{4}{9}q+\frac{1}{9}q}{3}$, *i. e.* $\frac{5}{27}q$: now it's evident the Divisor 3 is the number of the Gamesters.

To find what is due to one in any case; *viz.* if either he, or any of his Fellow-Gamesters win the following Game; we must consider first the more simple Cases, and by their help the following; for as this Case could not be solv'd before the Case of the eighth Proposition was calculated, in which, the Games wanting were 1, 1, 2; so the Case, where the Games wanting are 1, 2, 3, cannot be calculated, without the Calculation of the Case, where the Games wanting are 1, 2, 2, (which we have just now perform'd) and likewise of the Case, where the Games wanting are 1, 1, 3, which can be done by the eighth: And by this means you may reckon all the Cases comprehended in the following Tables, and an infinite number of others.

Games

Games wanting	1,1,2	1,2,2,	1, 1,3,	1,2,3,
Their Shares	4,4,1,	17,5,5.	13,13.1,	19,6,2,
	9	27	27	27

Games wanting	1, 1,4	1, 1,5	1, 2,4	1, 2,5
Shares	40,40,1	121,121,1	178,58,7	542,179,8
	81	243	243	729

Games wanting	1,3,3	1, 3, 4	1, 3, 5
Their Shares	65,8,8	616,82,31	629,87,13
	81	729	729

Games wanting	2, 2, 3	2, 2, 4	2, 2, 5
Their Shares	34,34,13	338,338,53	353.353.23
	81	729	729

Games wanting	2. 2. 3	2, 3, 4	2, 3, 5
Their Shares	133,55,55	451,195,83	433,635,119
	243	729	1187

As for the *Dice*; theſe Queſtions may be propoſed, at how many Throws one may wager to throw 6, or any Number below that, with one Die; How many Throws are required for 12 upon two Dice; or 18 on 3; and ſeveral other Queſtions to this purpoſe.

For the reſolving of which, it muſt be conſider'd, that in one Die there are ſix different Throws, all equally probable to come up; for I ſuppoſe the Die has the exact

exact figure of a Cube: On two Dice there are 36 different throws; for in respect to every throw of one Die, any one throw of the 6 of the other Die may come up; and 6 times 6 make 36. In three Dice there are 216 different throws; for in relation to any of the 36 throws of two Dice, any one of the six of the third may come up; and 6 times 36 make 216: So in four Dice there are 6 times 216 throws, that is, 1296: And so forward you may reckon the throws of any number of Dice, taking always, for the Addition of a new Die, 6 times the number of the preceding.

Besides, it must be observ'd, that in two Dice there is only one way 2 or 12 can come up; two ways that 3 or 11 can come up; for if I shall call the Dice A and B, to make 3 there may be 1 in A, and 2 in B, or 2 in A, and 1 in B; so to make 11, there may be 5 in A, and 6 in B, or 6 in A, and 5 in B; for 4 there are three Chances, 3 in A, and 1 in B, 3 in B, and 1 in A, or 2 as well in A as B; for 10 there are likewise three Chances; for 5 or 9 there are four Chances; for 6 or 8 five Chances; for 7 there are six Chances.

In 3 Dice there are found for

3	or	18	1
4	or	17	3
5	or	16	6
6	or	15	10
7	or	14	15
8	or	13	21
9	or	12	25
10	or	11	27

PROP. X.

To find at how many times one may undertake to throw 6 with one Die.

IF any ſhould undertake to throw 6 the firſt time, it's evident there's one Chance gives him the Stake, and five which give him nothing; for there are 5 throws againſt him, and only one for him. Let the Stake be call'd a, then he has one Expectation to gain a, and five to gain nothing, which, by the third Propoſition, is worth $\frac{1}{6}a$, and there remains for the other $\frac{5}{6}a$; ſo he who undertakes, with one Die, to throw 6 the firſt time, ought to wager only 1 to 5.

2. Suppoſe one undertake, at two Throws of 1 Die, to throw 6, his Hazard is calculated thus; if he throw 6 at the firſt, he has a the Stake; if he do not, there re-

mains to him one throw, which, by the former Caſe, is worth $\frac{1}{6}a$; but there is but one Chance which gives him 6 at the firſt throw, and five Chances againſt him; ſo there is one Chance which gives him a, and five which give him $\frac{1}{6}a$, which by the ſecond Propoſition, is worth $\frac{11}{36}a$, ſo there remains to his Fellow-Gameſter $\frac{25}{36}a$; ſo the Value of my Expectation to his, is as 11 to 25, *i. e.* leſs than 1 to 2.

By the ſame method of Calculation, you will find, that his hazard who undertakes to throw 6 at three times with one Die, is $\frac{91}{216}a$; ſo that he can only lay 91 againſt 125, which is ſomething leſs than 3 to 4.

He who undertakes to do it at four times, his hazard is $\frac{671}{1296}a$, ſo he may wager 671 againſt 625, that is, ſomething more than 1 to 1.

He who undertakes to do it at five times, his hazard is $\frac{4651}{7776}a$, ſo he can wager 4651 againſt 3125, that is, ſomething leſs than 3 to 2.

His hazard who undertakes to do it at 6 times, is $\frac{31031}{46656}a$, and he can wager 31031 againſt 15625, that is, ſomething leſs than 2 to 1.

Thus any number of throws may be eaſily found; but the following Propoſition will ſhew you a more compendious way of Calculation.

PROP.

PROP. XI.

To find at how many times one may undertake to throw 12 *with two Dice.*

IF one should undertake it at one throw, it's clear he has but one Chance to get the Stake a, and 35 to get nothing; which, by the third Proposition, is worth $\frac{1}{36}a$.

He who undertakes to do it at twice, if he throw 12 the first time, gains a; if otherwise, then there remains to him one throw, which, by the former Case, is worth $\frac{1}{36}a$; but there is but one Chance which gives 12 at the first throw, and 35 Chances against him; so he has 1 Chance for a, and 35 for $\frac{1}{36}a$, which by the third Proposition is worth $\frac{71}{1296}a$, and there remains to his Fellow-Gamester $\frac{1225}{1296}a$.

From these it's easy to find the Value of his hazard, who undertakes it at four times, passing by his case who undertakes it at three times.

If he who undertakes to do it at four times throws 12 the first or second Cast, then he has a; if not, there remains two other throws, which, by the former Case, are worth $\frac{71}{1296}a$; but for the same reason, in his two first throws, he has 71 Chances which give him a, against 1225 Chances,

in which it may happen otherwise; therefore at first he has 71 Chances which give him a, and 1225 which give him $\frac{71}{1296}a$, which by the third Proposition is worth $\frac{178991}{1679616}a$, which shews that their hazards to one another are as 178991 to 1500625.

From which Cases it is easy to find the Value of his Expectation, who undertakes to do it at 8 times, and from that, his Case who undertakes to do it at 16 times; and from his Case who undertakes to do it at 8 times, and his likewise who undertakes to do it at 16 times; it is easy to determine his Expectation who undertakes it at 24 times: In which Operation, because that which is principally sought, is the number of throws, which makes the hazard equal on both sides, *viz.* to him who undertakes, and he who offers, you may, without any sensible Error, from the Numbers (which else would grow very great) cut off some of the last Figures. And so I find, that he who undertakes to throw 12 with two Dice, at 24 times, has some loss; and he who undertakes it at 25 times, has some advantage.

PROP.

PROP. XII.

To find with how many Dice one can undertake to throw two Sixes at the first Cast.

THIS is as much, as if one would know, at how many throws of one Die, he may undertake to throw twice ſix: now if any ſhould undertake it at two throws, by what we have ſhewn before, his hazard would be $\frac{1}{36}a$; he who would undertake to do it at 3 times, if his firſt throw were not 6, then there would remain two throws, each of which muſt be 6, which (as we have ſaid) is worth $\frac{1}{36}a$; but if the firſt throw be 6, he wants only one 6 in the two following throws, which by the tenth Propoſition, is worth $\frac{11}{36}a$: but ſince he has but one Chance to get 6 the firſt throw, and five to miſs it; he has therefore, at firſt, one Chance for $\frac{11}{36}a$, and five Chances for $\frac{1}{36}a$, which, by the third Propoſition, is worth $\frac{16}{216}a$, or $\frac{2}{27}a$; after this manner ſtill aſſuming 1 Chance more, you will find that you may undertake to throw two Sixes at 10 throws of one Die, or 1 throw of ten Dice, and that with ſome advantage.

PROP.

PROP. XIII.

If I am to play with another one Throw, on this condition, that if 7 comes up I gain, if 10 he gains; if it happens that we must divide the Stake, and not play, to find how much belongs to me, and how much to him.

BEcause of the 36 different Throws of the two Dice, there are six which give 7, and 3 which give 10, and 27 which equals the Game, in which case there is due to each of us $\frac{1}{2}a$: But if none of the 27 should happen, I have 6, by which I may gain a, and 3, by which I may get nothing, which by the third Proposition, is worth $\frac{2}{3}a$; so I have 27 Chances for $\frac{1}{2}a$, and 9 for $\frac{2}{3}a$, which, by the third Proposition, is worth $\frac{13}{24}a$, and there remains to my Fellow-Gamester $\frac{11}{24}a$.

PROP. XIV.

If I were playing with another by turns, with two Dice, on this condition, that if I throw 7 I gain, and if he throw 6 he gains, allowing him the first throw: To find the proportion of my Hazard to his.

SUppose I call the Value of my Hazard x, and the Stakes a, then his Hazard will be $a - x$; then whenever it's his turn to throw, my Hazard is x, but when it's mine, the Value of my Hazard is greater. Suppose I then call it y; now because of the 36 throws of two Dice, there are five which give my Fellow-Gamester 6, thirty-one which bring it again to my turn to throw, I have five Chances for nothing, and thirty-one for y, which, by the third Proposition, is worth $\frac{31}{36}y$; but I suppos'd at first my Hazard to be x; therefore $\frac{31}{36}y = x$, and consequently $y = \frac{36}{31}x$. I suppos'd likewise, when it was my turn to throw, the Value of my Hazard was y; but then I have six Chances which give me 7, and consequently the Stake, and thirty which give my Fellow the Dice, that is, make my Hazard worth x; so I have six Chances for a, and thirty for x, which,

which, by Proposit. 3. is worth $\frac{6a + 30x}{36}$; but this by supposition is equal to y, which is equal (by what has been prov'd already) to $\frac{36}{31}x$; therefore $\frac{30x + 6a}{36} = \frac{36}{31}x$, and consequently $x = \frac{31}{61}a$, the Value of my Hazard, and that of my Fellow-Gamester is $\frac{30}{61}a$, so that mine is to his as 31 to 30.

Here follow some Questions which serve to exercise the former Rules.

1. *A* and *B* play together with two Dice, *A* wins if he throws 6, and *B* if he throws 7; *A* at first gets one throw, then *B* two, then *A* two, and so on by turns, till one of them wins. I require the proportion of *A*'s Hazard to *B*'s? *Answer*, It is as 10355 to 12276.

2. Three Gamesters, *A*, *B*, and *C*, take 12 Counters, of which there are four white and eight black; the Law of the Game is this, that he shall win, who, hood-wink'd, shall first chuse a white Counter; and that *A* shall have the first choice, *B* the second, and *C* the third, and so, by turns, till one of them win. *Quær.* What is the proportion of their Hazards?

3. *A* wagers with *B*, that of 40 Cards, that is, 10 of every Suit, he will pick out four, ſo that there ſhall be one of every ſuit. *A*'s Hazard to *B*'s in this caſe is as 1000 to 8139.

4. Suppoſing, as before, 4 white Counters and 8 black, *A* wagers with *B*, that out of them he ſhall pick 7 Counters, of which there are 3 white. I require the proportion of *A*'s Hazard to *B*'s?

5. *A* and *B* taking 12 Counters, each play with three Dice after this manner, that if 11 comes up, *A* ſhall give one Counter to *B*; but if 14 comes up, *B* ſhall give one to *A*, and that he ſhall gain who firſt has all the Counters. *A*'s Hazard to *B*'s is 244140625 to 282429536481.

The *Calculus* of the preceding Problems is left out by Monſ. *Huygens*, on purpoſe that the ingenious Reader may have the ſatisfaction of applying the former Method himſelf; it is in moſt of them more laborious than difficult: for *Example*, I have pitch'd upon the ſecond and third, becauſe the reſt can be ſolv'd after the ſame Method.

PROBLEM 1.

The first Problem is solv'd by the Method of Prop. 14. only with this difference, that after you have found the Share due to *B*, if *A* were to get no first throw, you must subtract from it $\frac{5}{36}$ of the Stake which is due to *A* for his Hazard of throwing six at the first throw.

PROBLEM 2.

As for the second Problem, it is solv'd thus; Suppose *A*'s Hazard, when it is his own turn to chuse, be x, when it is *B*'s, be y, and when it is *C*'s, be z; it is evident, when out of 12 Counters, of which there are 4 white and 8 black, he endeavours to chuse a white one, he has four Chances to get it, and eight to miss it; that is, he has four Chances to get the Stake a, and eight to make his hazard worth y: so $x = \frac{4a + 8y}{12}$, and consequently $y = \frac{12x - 4a}{8}$. When it is *B*'s turn to chuse, then he has four Chances for nothing, and eight for z, (that is, to bring it to *C*'s turn) consequently $y = \frac{8}{12}z = \frac{12x - 4a}{8}$; this Equation reduc'd gives $z = \frac{9x - 3a}{4}$; when it comes

comes to *C*'s turn to chuſe, then *A* has four Chances for nothing, and eight for x, conſequently $z = \frac{8}{12}x$, therefore $\frac{8}{12}x = \frac{9x - 3a}{4}$; this equation reduc'd gives $x = \frac{9}{19}a$, and conſequently there remains to *B* and *C* $\frac{10}{19}a$, which muſt be ſhar'd after the ſame manner, that is, ſo that *B* have the firſt choice, *C* the next, and ſo on, till one of them gain; the reaſon is, becauſe it had been juſt in *A* to have demanded $\frac{9}{19}$ of the Stake for not playing, and then the Seniority fell to *B*; now $\frac{10}{19}a$, parted betwixt *B* and *C*, by the former Method, gives $\frac{6}{19}$ to *B*, and $\frac{4}{19}$ to *C*; ſo *A*, *B*, and *C*'s Hazards from the beginning were as 9, 6, 4.

I have ſuppos'd here the Senſe of the Problem to be, that when any one chus'd a Counter, he did not diminiſh their number; but if he miſs'd of a white one, put it in again, and left an equal hazard to him who had the following choice; for if it be otherwiſe ſuppos'd, *A*'s ſhare will be $\frac{55}{123}$, which is leſs than $\frac{9}{19}$.

Prob. 2. It is evident, that wagering to pick out 4 Cards out of 40, ſo that there be one of every Suit, is no more than wagering, out of 39 Cards to take 3 which ſhall be of three propoſed Suits; for it is all one which Card you draw firſt, all the

hazard being, whether out of the 39 remaining you take 3, of which none ſhall be of the Suit you firſt drew: Suppoſe then you had gone right for three times, and were to draw your laſt Card, it is clear that there are 27 Cards, (*viz.* of the Suits you have drawn before) of which, if you draw any you loſe, and 10, of which if you draw any, you have the Stake a; ſo you have 10 Chances for a, and 27 for nothing, which, by Prop. 3. is worth $\frac{10}{37}a$. Suppoſe again you had gone right only for two Draughts, then you have 18 Cards (of the Suits you have drawn before) which make you loſe, and 20, which put you in the Caſe ſuppos'd formerly, *viz.* where you have but one Card to draw, which, as we have already calculated, is worth $\frac{10}{37}a$; ſo you have 18 Chances for nothing, and 20 for $\frac{10}{37}a$, which, by Prop. 3. is worth $\frac{100}{703}a$. Suppoſe again you have 3 Cards to draw, then you have 9 (of the Suit you drew firſt) which make you loſe, and 30 which put you in the caſe ſuppos'd laſt; ſo you have 9 Chances for nothing, and 30 for $\frac{100}{703}a$, which by Propoſ. 3. is worth $\frac{3000}{27417}a$, or $\frac{1000}{9139}a$, and you leave to your Fellow-Gameſter $\frac{8139}{9139}a$; ſo your hazard is to his as 1000 to 8139.

It is eaſy to apply this Method to the Games that are in uſe amongſt us: For *Example,*

Example, If *A* and *B*, playing at *Backgammon*, *A* had already gain'd one end of three, and *B* none, and if *A* had the Dice in his Hand for the laſt throw of the ſecond end, all his Men but two upon the Ace Point being already caſt off: *Quær.* What is the proportion of *A*'s Hazard to *B*'s?

Solution: There being of the 36 Throws of two Dice, ſix which give Doublets; if *A* throw any of the ſix, he has the ſtake a; if he throw any of the other thirty, then he wants but one Game, and his Fellow-Gameſter three, which, by Prop. V. is $\frac{7}{8}a$; ſo *A* has ſix Chances for a, and thirty for $\frac{7}{8}a$, which, by Prop. III. is worth $\frac{129}{144}a$, and there remains to his Play-Fellow $\frac{15}{144}a$; ſo *A*'s Hazard to *B*'s is as 129 to 15, that is, leſs than 9 to 1.

Suppoſing the ſame Caſe, and if their Bargain had been, that he who gain'd three ends before the other gain'd one, ſhould have double of what each ſtak'd, that is, the Stake and a half more, then there had been due to *A* $\frac{281}{288}$ of the Stake, that is, *B* ought only to take $\frac{1}{48}$, and leave the reſt to *A*.

Thus likewiſe, if you apply the former Rule to the *Royal-Oak Lottery*, you will find, that he who wagers that any Figure ſhall come up at the firſt throw, ought to wager 1 againſt 31; that he who wagers it

it ſhall come up at one of two throws, ought to wager 63 againſt 961; that he who wagers that a Figure ſhall come up at once in three times, ought to lay 125055 againſt 923521, *&c.* it being only ſomewhat tedious to calculate the reſt. Where you will find, that the equality will not fall as ſome imagine on 16 Throws, no more than the equality of wagering at how many Throws of one Die 6 ſhall come up, falls on three; the contrary of which you have ſeen already demonſtrated: you will find by calculation, that he has the Diſadvantage, who wagers, that 1 of the 32 different Throws of the *Royal-Oak Lottery* ſhall come at once of 20 times, and that he has ſome advantage, who wagers on 22 times, ſo the neareſt to Equality is on 21 times. But it muſt be remembred, that I have ſuppos'd in the former Calculation, the Ball in the *Royal-Oak Lottery* to be regular, tho' it can never be exactly ſo; for he, who has the ſmalleſt Skill in Geometry, knows, that there can be no regular Body of 32 ſides; and yet this can be of no Advantage to him who keeps it.

To find the Value of the Throws of Dice, as to the Quantity.

NOthing is more easy, than by the former Method to determine the Value of any number of Throws of any number of Dice; for in one throw of a Die, I have an equal Chance for 1, 2, 3, 4, 5, 6, consequently my Hazard is worth their Sum 21 divided by their Number 6, that is, $3\frac{1}{2}$. Now if one throw of a Die be worth $3\frac{1}{2}$, then two throws of a Die, or one throw of two Dice is worth 7, two throws of two Dice, or one throw of four Dice is worth 14, *&c.* The general Rule being to multiply the Number of Dice, the Number of Throws, and $3\frac{1}{2}$ continually.

This is not to be understood as if it were an equal wager to throw 7, or above it, with two Dice at one throw; for he who undertakes to do so, has the Advantage by 21 against 15. The meaning is only, if I were to have a Guinea, a Shilling, or any thing else, for every Point that I threw with

with two Dice at one throw, my Hazard is worth 7 of theſe, becauſe he who gave me 7 for it, would have an equal probability of gaining or loſing by it, the Chances of the Throws above 7, being as many as of theſe below it: So it is more than an equal wager to throw 14 at leaſt at two Throws of two Dice, becauſe it is more probable that 14 will come, than any one number beſides, and as probable that it will be above it as below it; but if one were to buy this Hazard at the rate above-mention'd, he ought juſt to give 14 for it. The equal wager in one Throw of two Dice, is to throw 7 at leaſt one time, and 8 at leaſt another time, and ſo *per vices:* The reaſon is, becauſe in the firſt Caſe I have 21 Chances againſt 15, and in the ſecond 15 Chances againſt 21.

Of RAFFLING.

IN *Raffling*, the different Throws and their Chances are these; Where it is to be observed, that of the 216 different Throws of three Dice, there are only 96 that give Doublets, or two, at least, of a kind; so it is 4 to 5 that with three Dice you shall throw Doublets, and it is 1 to 35 that you throw a Raffle, or all three of a kind. It is evident likewise, that it is an even wager to throw 11 or above it, because there are as many Chances for 11, and the Throws above it, as for the Throws below it; but tho' it be an even wager to throw 11 at one Throw, it is a disadvantage to wager to throw 22 at two Throws, and far more to wager to throw 33 at three Throws; and yet it is more than an equal Wager that you shall throw 21 at two Throws in Raffling, because it is as probable that you will, as that you will not throw 11, at least, the first time, and

Throws.		Chan.
3	18	1
4	17	3
5	16	6
6	15	4
7	14	9
8	13	9
9	12	7
10	11	9

more than probable that you will throw 10, at leaſt, the ſecond time.

For an inſtance of the plainneſs of the preceding Method, I will ſhew, how by ſimple Subtraction, the moſt part of the former Problems may be ſolv'd.

Suppoſe *A* and *B*, playing together, each of them ſtakes 32 Shillings, and that *A* wants one Game of the number agreed on, and *B* wants two; to find the ſhare of the Stakes due to each of them. It's plain, if *A* wins the next Game, he has the whole 64 Shillings; if *B* wins it, then their Shares are equal; therefore ſays *A* to *B*, If you will break off the Game, give me 32, which I am ſure of, whether I win or loſe the next Game; and ſince you will not venture for the other 32, let us part them equally, that is, give me 16, which, with the former 32 make 48, leaving 16 to you.

Suppoſe *A* wanted one Game, and *B* three; if *A* wins the next Game, he has the 64 Shillings; if *B* wins it, then they are in the condition formerly ſuppos'd, in which caſe there is 48 due to *A*; therefore ſays *A* to *B*, give me the 48 which I am ſure of, whether I win or loſe the next Game; and ſince you will not hazard for the other 16, let us part them equally, that is, give me 8, which, with the former 48, make

make 56, leaving 8 to you; and ſo all the other Caſes may be ſolv'd after the ſame manner.

Suppoſe *A* wagers with *B*, that with one Die he ſhall throw 6 at one of three Throws, and that each of them ſtakes 108 Guineas; to find what is the proportion of their Hazards. Now there being in one Throw of a Die but one Chance for 6, and five Chances againſt it, one Throw for 6 is worth $\frac{1}{6}$ of the Stake; therefore ſays *B* to *A*, of the 216 Guineas take a ſixth part for your firſt Throw, that is, 36; for your next Throw take a ſixth part of the remaining 180, that is, 30; and for your third Throw, take a ſixth part of the remaining 150, that is, 25, which in all make 91, leaving to me 125; ſo his hazard who undertakes to throw 6 at one of three Throws, is 91 to 125.

Suppoſe *A* had undertaken to throw 6 with one Die at one Throw of four, and that the whole Stake is 1296; ſays *A* to *B*, every Throw for 6 of one Die, is worth the ſixth part of what I throw for; therefore for my firſt Throw give me 216, which is the ſixth part of 1296, and there remains 1080, I muſt have the ſixth part of that, *viz.* 180, for my ſecond Throw; and the ſixth part of the remaining 900, which is 150, for my third Throw; and

the ſixth part of the laſt remainder 750, which is 125 for my fourth; all this added together makes 671, and there remains to you 625; ſo it is evident, that *A*'s Hazard, in this Caſe, is to *B*'s 671 to 625.

Suppoſe *A* is to win the Stakes (which we ſhall ſuppoſe to be 36) if he throws 7 at once of twice with two Dice, and *B* is to have them if he does not; ſays *B* to *A*, the Chances which give 7 are 6 of the 36, which is as much as 1 of 6; therefore for your firſt Throw you ſhall have a ſixth part of the 36, which is 6; and for your next Throw a ſixth part of the remainder 30, which is 5; this in all makes 11; ſo you leave 25 to me; ſo *A*'s Hazard is to *B*'s as 11 to 25.

It were eaſy, at this rate, to calculate the moſt intricate Hazards, were it not that Fractions will occur; which, if they be more than $\frac{1}{2}$, may be ſuppos'd equal to an Unit, without cauſing any remarkable Error in great Numbers.

It will not be amiſs, before I conclude, to give you a Rule for finding in any number of Games the Value of the firſt, becauſe *Huygens*'s Method, in that caſe, is ſomething tedious.

Suppoſe *A* and *B* had agreed, that he ſhould have the Stakes who did win the firſt 9 Games, and *A* had already won one of

of the 9; I would know what ſhare of *B*'s Money is due to *A* for the Advantage of this Game. To find this, take the firſt eight even Numbers 2, 4, 6, 8, 10, 12, 14, 16, and multiply them continually, that is, the firſt by the ſecond, the product by the third, *&c.* take the firſt eight odd Numbers 1, 3, 5, 7, 9, 11, 13, 15, and do juſt ſo by them, the product of the even Number is the Denominator, and the product of the odd Number the Numerator of a Fraction, which expreſſeth the quantity of *B*'s Money due to *A* upon the winning of the firſt Game of 9; that is, if each ſtak'd a number of Guineas, or Shillings, *&c.* expreſs'd by the product of the even Numbers, there would belong to *A*, of *B*'s Money, the Number expreſs'd by the product of the odd Numbers. For *Example*, Suppoſe *A* had gain'd one Game of 4, then by this Rule, I take the three firſt even Numbers, 2, 4, 6, and multiply them continually, which make 48, and the firſt three odd Numbers, 1, 3, 5, and multiply them continually, which make 15; ſo there belongs to *A* $\frac{15}{48}$ of *B*'s Money, that is, if each ſtak'd 48, there would belong to *A*, beſides his own, 15 of *B*'s. Now by *Huygens*'s Method, if *A* wants but three Games while *B* wants four, there is due to *A* $\frac{21}{32}$ of the Stake; by this Rule there is due

to

to *A* $\frac{15}{48}$ of *B*'s Money, which is $\frac{15}{49}$ of the Stake, which, with his own $\frac{48}{96}$ of the Stake, makes $\frac{62}{96}$ or $\frac{21}{32}$ of the Stake; and ſo in every Caſe you will find *Huygens*'s Method, and this will give you the ſame Number: A Demonſtration of it you may ſee in a Letter of Monſieur *Paſcal*'s to Monſieur *Fermat*; though it be otherwiſe expreſs'd there than here, yet the conſequence is eaſily ſupply'd. To prevent the labour of Calculation, I have ſubjoin'd the following Table, which is calculated for two Gameſters, as Monſieur *Huygens*'s is for three.

If each of us ſtake 256 *Guineas* in

There belongs to me 256 of my Play-fellow

	6	5	4	3	2	1
1*ſt* *Game*	63	70	80	96	128	256
2 1*ſt* *Games*	126	140	160	192	256	
3 1*ſt* *Games*	182	200	224	256		
4 1*ſt* *Games*	224	240	256			
5 1*ſt* *Games*	248	256				
6 1*ſt* *Games*	256					

The

The Uſe of the Table is plain; for let our Stakes be what they will, I can find the Portion due to me upon the winning the firſt, or the firſt two Games, *&c.* of 2, 3, 4, 5, 6. For *Example,* If each of us had ſtak'd 4 Guineas, and the number of Games to be play'd were 3, of which I had gain'd 1, ſay, As 256 is to 96, ſo is 4 to a fourth.

$$256 : 96 :: 4 : 1\tfrac{1}{2}.$$

To find what is the Value of his Hazard, who undertakes, at the firſt Throw, to caſt Doublets, in any given number of Dice.

In two Dice it is plain, to avoid Doublets, every one of the ſix different Throws of the firſt, can only be combin'd with five of the ſecond, becauſe one of the ſix is of the ſame kind, and conſequently makes Doublets; for the ſame reaſon, the thirty Throws of two Dice, which are not Doublets, can only be combin'd with four Throws of a third Dice, and three Throws of a fourth Dice; ſo generally it is this Series,

$$\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 0, \&c.}{6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6, \&c.}$$

The ſecond Series is the Sum of the Chances, and the firſt the Number of Chances againſt him who undertakes to throw Doublets, each Series to be continu'd ſo many terms, as are the number of Dice. For *Example*, If one ſhould undertake to throw Doublets at the firſt Throw of four Dice, his Adverſary's Hazard is $\frac{6 \times 5 \times 4 \times 3}{6 \times 6 \times 6 \times 6} = \frac{360}{1296}$ or $\frac{5}{18}$, leaving to him $\frac{13}{18}$, ſo he has 13 to 5. In ſeven Dice, you ſee the Chances againſt him are 0, becauſe then there muſt neceſſarily be Doublets.

Of

Of WHIST.

If there be four playing at Whist, *it is* 15 *to* 1 *that any two of them shall not have the four Honours, which I demonstrate thus.*

SUppose the four Gamesters be *A*, *B*, *C*, *D*: If *A* and *B* had, while the Cards are a dealing, already got three Honours, and wanted only one, since it is as probable that *C* and *D* will have the next Honour, as *A* and *B*; if *A* and *B* had laid a Wager to have it, there is due to them but $\frac{1}{2}$ of the Stake: If *A* and *B* wanted two of the four, and had wager'd to have both those two, then they have an equal hazard to get nothing, if they miss the first of these two, or to put themselves in the former Case if they get; so they have an equal Hazard to get nothing, or $\frac{1}{2}$, which, by *Prop.* 1. is worth $\frac{1}{4}$ of the Stake; so if they want three Honours, you will find due to them $\frac{1}{8}$ of the Stake; and if they wanted four, $\frac{1}{16}$ of the Stake, leaving *C* and *D* $\frac{15}{16}$; so *C* and *D* can wager 15 to 1, that

A and *B* ſhall not have all the four Honours.

It is 11 *to* 5 *that* A *and* B *ſhall not have three of the four Honours, which I prove thus:*

It is an even Wager, if there were but three Honours, that *A* and *B* ſhall have two of theſe three, ſince 'tis as probable that they will have two of the three, as that *C* and *D* ſhall have them; conſequently, if *A* and *B* had laid a Wager to have two of three, there is due to them $\frac{1}{2}$ of the Stake. Now ſuppoſe *A* and *B* had wager'd to have three of four, they have an equal hazard to get the firſt of the four, or miſs it; if they get it, then they want two of the three, and conſequently there is due to them $\frac{1}{2}$ of the Stake; if they miſs it, then they want three of the three, and conſequently there is due to them $\frac{1}{8}$ of the Stake; therefore, by *Prop.* 1. their Hazard is worth $\frac{5}{16}$, leaving to *C* and *D* $\frac{11}{16}$.

A *and* B *playing at* Whiſt *againſt* C *and* D; A *and* B *have eight of ten, and* C *and* D *nine, and therefore can't reckon Honours; to find the proportion of their Hazards.*

There

There is $\frac{5}{16}$ due to *A* and *B* upon their hazard of having three of four Honours; but ſince *C* and *D* want but one Game, and *A* and *B* two, there is due to *A* and *B* but $\frac{1}{4}$, or $\frac{4}{16}$ more upon that account, by *Prop.* 4. this in all makes $\frac{9}{16}$, leaving to *C*, and *D* $\frac{7}{16}$; ſo the hazard of *A* and *B* to that of *C* and *D*, is as 9 to 7.

In the former Calculations I have abſtracted from the ſmall difference of having the Deal, and being Seniors.

All the former Caſes can be calculated by the *Theorems* laid down by Monſieur *Huygens*; but Caſes more compos'd require other Principles: for the eaſy and ready Computation of which, I ſhall add one *Theorem* more, demonſtrated after Monſieur *Huygens*'s Method.

THEOREM.

If I have p Chances for a, q Chances for b, and r Chances for c, then my hazard is worth $\frac{ap+bq+cr}{p+q+r}$; that is, a multiplied into the number of its Chances added to b, multiplied into the number of its Chances, added to c, multiplied into the number of its Chances, and the Sum divided by the Sum of Chances of a, b, c.

To investigate as well as demonstrate this *Theorem*, suppose the value of my hazard be x, then x must be such, as having it, I am able to purchase as good a hazard again in a just and equal Game. Suppose the Law of it be this, That playing with so many Gamesters as, with myself, make up the number $p + q + r$, with as many of them as the number p represents, I make this bargain, that whoever of them wins shall give me a, and that I shall do so to each of them if I win; with the Gamesters represented by the number of q, I bargain to get b, if any of them win, and to give b to each of them, if I win my self; and with the rest of the Gamesters, whose number is $r - 1$, I bargain to give, or to get c after the same manner: Now all being in an equal probability to gain, I have p Chances to get a, q Chances to get b, and $r - 1$ Chances to get c, and one Chance, *viz.* when I win myself, to get $px + qx + rx - ap - bq - rc + c$, which, if it be suppos'd equal to c, then I have p Chances for a, q Chances for b, and r Chances for c (for I had just now $r - 1$ Chances for it) and therefore, in case $px + qx + rx - ap - bq - rc + c = c$, then is $x = \frac{ap + bq + cr}{p + q + r}$.

By

By the ſame way of reaſoning you will find, If I have p Chances for a, q Chances for b, r Chances for c, and s Chances for d, that my hazard is $\frac{ap+bq+cr+ds}{p+q+r+s}$, &c.

In NUMBERS.

If I had two Chances for 3 Shillings, four Chances for 5 Shillings, and one Chance for 9 Shillings, then, by this Rule, my hazard is worth 5 Shillings; for $\frac{2\times3+4\times5+1\times9}{7}=5$; and it is eaſy to prove, that with five Shillings I can purchaſe a like hazard again; for ſuppoſe I play with ſix others, each of us ſtaking 5 Shillings; with two of them I bargain, that if either of them win, he muſt give me 3 Shillings, and that I ſhall do ſo to them; and with the other four I bargain juſt ſo, to give or to get 5 Shillings: This is a juſt Game, and all being in an equal probability to win; by this means I have two Chances to get 3 Shillings, four Chances to get 5 Shillings, and one Chance to get 9 Shillings, *viz.* when I win my ſelf; for then out of the Stake, which makes 35 Shillings, I muſt give the firſt two 6 Shillings, and the other four 20 Shillings, ſo there remains juſt 9 to myſelf.

It is eaſy, by the help of this *Theorem*, to calculate in the Game of Dice, commonly call'd *Hazard*, what Mains are beſt to ſet on, and who has the Advantage, the Caſter or Setter. The *Scheme* of the Game, as I take it, is thus ;

	Throws next following for	
Mains.	The *Caſter*.	The *Setter*.
V.	V.	II. III. XI. XII.
VI.	VI. XII.	XI. II. III.
VII.	VII. XI.	XII. II. III.
VIII.	VIII. XII.	XI. II. III.
IX.	IX.	II. III. XI. XII.

By an eaſy Calculation you will find, if the Caſter has IV, and the Setter VII, there is due to the Caſter $\frac{1}{3}$ of the Stake ; if he has

V againſt VII, $\frac{2}{5}$ of the Stake.
VI againſt VII, $\frac{5}{11}$ of the Stake.
IV againſt VI, $\frac{3}{8}$ of the Stake.
V againſt VI, $\frac{4}{9}$ of the Stake.
IV againſt V, $\frac{3}{7}$ of the Stake.

I need not tell the Reader, that IV is the ſame with X, V with IX, and VI with VIII.

Suppoſe

Suppoſe then VII be the Main: To find the proportion of the hazard of the Caſter to that of the Setter.

By the Law of the Game, the Caſter, before he throws next, has four Chances for nothing, *viz.* theſe II, III, XII; eight Chances for the whole Stake, *viz.* thoſe of VII, XI; ſix Chances for $\frac{1}{3}$, *viz.* thoſe IV, X; eight Chances for $\frac{2}{5}$, *viz.* thoſe of V, IX; and ten Chances for $\frac{5}{11}$, *viz.* theſe of VI, VIII; ſo his hazard, by the preceding *Theorem*, is

$$\frac{4\times0+8\times1+6\times\frac{1}{3}+8\times\frac{2}{5}+10\times\frac{5}{11}}{36}$$

Now to ſave the trouble of a tedious reduction, ſuppoſe the Stake which they play for be 36, that is, the Setter had laid down 18; in that caſe, every one of theſe Fractions are ſo many parts of an Unit, which being gather'd into one Sum, give $17\frac{41}{55}$ to the Caſter, leaving $18\frac{14}{55}$ to the Setter; ſo the hazard of the Caſter is to that of the Setter 244, 251.

Suppoſe VI, or VIII, be the Main, then the Share of the Caſter is

II.
III. VI. IV. V.
XI. XII. X. IX. VIII. VII.

$5\times0+6\times1+6\times\frac{3}{8}+8\times\frac{4}{9}+5\times\frac{1}{2}+6\times\frac{6}{11}=17\frac{229}{396}$,

leaving

leaving to the Setter $18\frac{167}{396}$; so the hazard of the Caster is to that of the Setter as 6961 to 7295.

Suppose V, or IX be the Main, then the Share of the Caster is

II.					
III.					
XI.		IV.		VI.	
XII.	V.	X.	IX.	VIII.	VII.

$$6\times 0+4\times 1+6\times\tfrac{3}{7}+4\times\tfrac{1}{2}+10\times\tfrac{5}{9}+6\times\tfrac{3}{5}=17\tfrac{229}{315},$$

leaving to the Setter $18\frac{86}{315}$; so the hazard of the Caster is to that of the Setter as 1396 to 1439.

It is plain, that in every case the Caster has the Disadvantage, and that V, or IX, are better Mains to set on than VII, because, in this last Cast, the Setter has but 18 and $\frac{14}{55}$, or $\frac{84}{330}$; whereas, when V or IX is the Main, he has $18\frac{86}{315}$; likewise VI, or VIII, are better Mains than V, or IX, because $\frac{167}{396}$ is a greater Fraction than $\frac{86}{315}$.

All those Problems suppose Chances, which are in an equal probability to happen; if it should be suppos'd otherwise, there will arise variety of Cases of a quite different

different nature, which, perhaps, 'twere not unpleaſant to conſider: I ſhall add one *Problem* of that kind, leaving the Solution to thoſe who think it merits their pains.

In Parallelipipedo cujus latera ſunt ad invicem in ratione a, b, c: *Invenire quotâ vice quivis ſuſcipere poteſt, ut datum quodvis planum, v.g.* a b *jaciat.*

A DEMONSTRATION

OF THE

Gain of the BANKER

In any Circumſtance of the

Game call'd *Pharaon*,

And how to determine the

Odds at the ACE of HEARTS, or FAIR CHANCE;

With the ARITHMETICAL SOLUTION of ſome Queſtions relating to LOTTERIES:

And a few Remarks upon HAZARD and BACKGAMMON.

Of the Game of PHARAON.

IN order to demonstrate the Odds or Gain of the Banker in any circumstance of the Game of *Pharaon*, it will be necessary to solve the following Problem.

PROB. I.

Two Persons A, B, *out of a heap of* 9 *Cards, four of which are red and five black, undertake to draw a red one blindfold, and he shall be reputed to win that draws the first: Now supposing* A *to have the first choice,* B *the second,* A *the third, and so on by turns till one of them wins;* Quære, *The proportion of their Hazards?*

SOLUTION.

Let n be the number of all the Cards, r the number of red ones, b the number of black

black ones, and 1 the whole Stake, or the Sum play'd for.

1°, Since A has r Chances for a red Card, and b Chances for a black one, it follows by the third Proposition that his Expectation is worth $\frac{r}{r+b}$ or $\frac{r}{n}$ of the Stake, equal to $\frac{r}{n} \times 1$; and accordingly let it be agreed between the two Gamesters, that instead of A's attempting to draw a red Card he shall actually take out a black one, and as an equivalent shall have $\frac{r}{n}$ paid him out of the Stake, which being done, there will remain $1 - \frac{r}{n} = \frac{n-r}{n} = \frac{b}{n}$.

2°, Since the remaining Cards are $n-1$, and B has r Chances for a red Card, it follows that his Probability of winning will be $\frac{r}{n-1}$, and consequently his Expectation upon the remaining Stake $\frac{b}{n}$ will be $\frac{r}{n-1} \times \frac{b}{n} = \frac{rb}{n \times \overline{n-1}}$. But instead of B's drawing, we will suppose the Sum $\frac{rb}{n \times \overline{n-1}}$ paid him out of the Stake, and that a black Card being taken out of the heap as before, then the remaining Stake will be $\frac{b}{n} - \frac{rb}{n \times \overline{n-1}}$ or $\frac{b \times \overline{n-1}}{n \times \overline{n-1}} - \frac{b \times \overline{n-b}}{n \times \overline{n-1}} = \frac{b \times \overline{b-1}}{n \times \overline{n-1}}$.

3°,

2

3°, Now it comes to *A*'s turn again to chuse; and since he has still r Chances for a red Card, and the number of remaining Cards are $n-2$, his Probability of winning this time will be $\frac{r}{n-2}$, and his Expectation upon the remaining Stake $\frac{b\times b-1}{n\times n-1}$ will be $\frac{b\times b-1}{n\times n-1}\times\frac{r}{n-2}$, which we will likewise suppose to be paid him out of the Stake, and then there will remain $\frac{b\times b-1}{n\times n-1}-\frac{b\times b-1\times r}{n\times n-1\times n-2}$, or $\frac{b\times b-1\times n-2}{n\times n-1\times n-2}-\frac{b\times b-1\times n-b}{n\times n-1\times n-2}=\frac{b\times b-1\times b-2}{n\times n-1\times n-2}$, out of which *B* may have $\frac{b\times b-1\times b-2\times r}{n\times n-1\times n-2\times n-3}$; and so we may proceed till the whole Stake is exhausted.

But I hope what has been said is sufficient to see the Law of Continuation, and form this general Series; *viz.* $\frac{r}{n}+\frac{b}{n-1}\text{P}+\frac{b-1}{n-2}\text{Q}+\frac{b-2}{n-3}\text{R}+\frac{b-3}{n-4}\text{S}$, *&c.* In which 'tis evident that P, Q, R, S, *&c.* denote the preceding Terms. Now to apply this Series to practice, we must take as many Terms of it, as there are Units in $b+1$; for since b represents the number of black Cards, the number of Drawings cannot exceed $b+1$; therefore take for *A* the first, third,

third, fifth, *&c.* Terms; and for *B*, the second, fourth, sixth, *&c.* Terms, and the Sum of those Terms will be the respective Expectations of *A*, *B*; or because the Stake is fix'd, these Sums will be proportional to their respective Probabilities of winning. For instance, if we apply this to the present Case, the general Series will be $\frac{4}{9} + \frac{5}{8}P + \frac{4}{7}Q + \frac{3}{6}R + \frac{2}{5}S + \frac{1}{4}T$. And to bring this Series into whole Numbers, let us assume x a whole Number, which multiply'd by $\frac{4}{9}$ shall be a whole Number $=$ P. Therefore since $\frac{4x}{9} = P$, $\frac{4x}{9} \times \frac{5}{8} = Q$, and $\frac{4x}{9} \times \frac{5}{8} \times \frac{4}{7} = R$, &c. each of which Terms are to be whole Numbers: Now 'tis evident that the Denominator of the Fraction represented by P, is an aliquot part of that represented by Q, and every Denominator an aliquot part of the next following, and so the last Denominator $9 \times 8 \times 7 \times 6 \times 5 \times 4$ is a Multiple to all the preceding Denominators; and consequently if each Numerator be multiplied by it, instead of x, and the Products divided by their respective Denominators, the Quotients will all be whole Numbers, this is universally true: But in this Case, if the last Fraction $\frac{4x \times 5 \times 4 \times 3 \times 2 \times 1}{9 \times 8 \times 7 \times 6 \times 5 \times 4}$ be reduced to its least Terms, *viz.* $\frac{x}{126}$, its Denominator 126 is the least

common

common Multiple to all the reſt of the Fractions when reduc'd to their leaſt Terms.

Which being determin'd, the Terms of the Series are eaſily found by the following Operation to be $56 + 35 + 20 + 10 + 4 + 1$.

OPERATION.

$$
\begin{array}{rl}
 & 126 \\
 & \underline{4} \\
 & 504 \\
9) & \overline{} \\
 & 56 = P \\
 & \underline{5} \\
 & 280 \\
8) & \overline{} \\
 & 35 = Q \\
 & \underline{4} \\
 & 140 \\
7) & \overline{} \\
 & \underline{20} = R
\end{array}
\qquad\qquad
\begin{array}{rl}
 & 20 = R \\
 & \underline{3} \\
 & 60 \\
6) & \overline{} \\
 & 10 = S \\
 & \underline{2} \\
 & 20 \\
5) & \overline{} \\
 & 4 = T \\
 & \underline{1} \\
 & 4 \\
4) & \overline{} \\
 & \underline{1} = U
\end{array}
$$

Wherefore aſſigning to *A* $56+20+4=80$
And to *B* - - - - - - - - $35+10+1=46$

And their Probabilities of winning will be as 80 to 46, or as 40 to 23. And if there be never ſo many Gameſters *A*, *B*, *C*, *D*, &c. the Probabilities of winning may as eaſily be aſſign'd by the general Series, as for theſe two.

REMARK.

The preceding Series may in any particular Case be easily shorten'd; for if $r=1$, then the Series will be

$$\frac{1}{n}\times\overline{1+1+1+1+1+1}, \text{ \&c.}$$

If $r=2$, then the Series will be $\frac{2}{n\times n-1}$ $\times\overline{n-1}+\overline{n-2}+\overline{n-3}+\overline{n-4}$, &c.

If $r=3$, then the Series will be $\frac{3}{n\times\overline{n-1}\times\overline{n-2}}$ $\times\overline{n-1}\times\overline{n-2}+\overline{n-2}\times\overline{n-3}+\overline{n-3}\times\overline{n-4}$ &c.

If $r=4$, then the Series will be $\frac{4}{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}+\overline{n-2}\times\overline{n-3}\times\overline{n-4}$ &c.

Wherefore rejecting the common Multiplicators, the several Terms of those Series taken in due order will be proportional to the several Expectations of any number of Gamesters. Thus in the Case of this Problem, where $n=9$, $r=4$, $b=5$, the Terms of the Series will be

For *A*.		For *B*.	
$8\times7\times6=$	336	$7\times6\times5=$	210
$6\times5\times4=$	120	$5\times4\times3=$	60
$4\times3\times2=$	24	$3\times2\times1=$	6
	480		276

Hence

Hence it follows, that the Probabilities of winning will be reſpectively as 480 to 276, or dividing both by 12, as 40 to 23, the ſame as before.

The Game of PHARAON.

Rules of the Play.

Firſt, The Banker holds a Pack of 52 Cards.

Secondly, He draws the Cards one after the other, laying them alternately to his right and left-hand.

Thirdly, The Ponte may, at his choice, ſet one or more Stakes, upon one or more Cards taken out of his parcel of 13 Cards, from the Ace to the King incluſive, call'd a Book; either before the Banker has begun to draw the Cards, or after he has drawn any number of Couples, which are commonly call'd Pulls.

Fourthly, The Banker wins the Stake of the Ponte, when the Card of the Ponte comes out in an odd place on his right-hand; but loſes as much to the Ponte when it comes out in an even place on his left-hand.

Fifthly, The Banker wins half the Ponte's Stake, when in the ſame pull the Card of the Ponte comes out twice.

Sixthly, When the Card of the Ponte, being once in the Stock, happens to be the laſt, the Ponte neither wins nor loſes.

Seventhly, The Card of the Ponte being but twice in the Stock, and the two laſt Cards happening to be his Cards, he then loſes his whole Stake.

Prob. II.

To find at Pharaon the Gain of the Banker, when any number of Cards remain in the Stock; having the number of times that the Ponte's Card is contain'd in it, given.

WHerefore this Problem admits of four Caſes, ſince the Ponte's Card may be contain'd either once, twice, thrice, or four times in the Stock.

Solution of the firſt Case.

The Banker, according to the Law of the Game, has the following number of Chances for winning and loſing; *viz.*

1	1 Chance for winning	1
2	1 Chance for loſing	1
3	1 Chance for winning	1
3	1 Chance for loſing	1
5	1 Chance for winning	1
*	1 Chance for loſing	0

Hence

Hence the Banker has one Chance more for winning than for losing, and the number of all the Chances are equal to n, the number of Cards in the Stock; therefore $\frac{1}{n}$ is the Gain of the Banker upon the Stake. To illustrate this Case,

Let it be required to find the Gain of the Banker, when there are 20 Cards remaining in the Stock, and the Ponte's Card but once in it, $\frac{1}{n} = \frac{1}{20}$.

Answer, He gains the twentieth part of his Stake.

Solution of the second CASE.

By the Remark belonging to the preceding Problem, it appears, that the Chances which the Banker has to win or lose his Stake would be proportional to these Numbers $n - 1$, $n - 2$, $n - 3$, &c. were the Banker not allow'd half the Stake upon drawing of Doublets, or the Ponte's Card twice together; upon which account the number of Chances represented for the Banker by $n - 1$ for winning, must be divided into two parts $n - 2$, and 1, whereof the first is proportional to the probability which the Banker has for winning the whole Stake of the Ponte, and the second is proportional to the probability of winning the half of it: for since the Banker is not intitled to the

whole Stake upon the Ponte's Card coming out in an odd place, till he knows whether the next Card be the Ponte's, his Chance for winning the whole Stake can be no greater than the Ponte's; becauſe there is the ſame reaſon, and eaſily prov'd, that the ſecond Card, and not the firſt, ſhould be the Ponte's, as the firſt Card, and not the ſecond. Wherefore the Chances that the Banker and the Ponte have to win the whole Stake are equal, with reſpect to the order of drawing, and conſequently ſhould be expreſs'd by the ſame number; whence ariſes this Scheme for determining the number of Chances which the Banker has for winning and loſing the Stake, and the number of Chances for winning the half Stake, repreſented by *y*.

SCHEME.

1	$\left\{\begin{matrix} n-2 \\ 1 \end{matrix}\right\}$	Chances for winning	$\left\{\begin{matrix} 1 \\ y \end{matrix}\right.$
2	$n-2$	Chances for loſing	1
3	$\left\{\begin{matrix} n-4 \\ 1 \end{matrix}\right\}$	Chances for winning	$\left\{\begin{matrix} 1 \\ y \end{matrix}\right.$
4	$n-4$	Chances for loſing	1
5	$\left\{\begin{matrix} n-6 \\ 1 \end{matrix}\right\}$	Chances for winning	$\left\{\begin{matrix} 1 \\ y \end{matrix}\right.$
6	$n-6$	Chances for loſing	1
*	1	Chance for winning	1

First, $\frac{n-2}{2}$ = Number of Chances for gaining y from the nature of the Scheme, and $n \times \frac{n-1}{2}$ = Sum of all the Chances. Therefore $\frac{n-2}{n \times n-1}$ is the probability of the Banker's gaining y, and $\frac{n-2}{n \times n-1} \times y$ = the Banker's Gain upon y.

Secondly, The Banker has but 1 Chance more to win the Stake than to lose; the rest for winning and losing being equal destroy each other. Therefore $\frac{1}{n \times \frac{n-1}{2}}$ is the probability of getting it, and $\frac{1}{n \times \frac{n-1}{2}} \times 1 =$ $\frac{2}{n \times n-1}$ is his Gain upon the Stake.

Therefore the Banker's Gain upon the whole is $\frac{n-2}{n \times n-1} \times y + \frac{2}{n \times n-1} = \frac{\overline{n-2} \times y + 2}{n \times n-1}$ $= \frac{\frac{1}{2}n+1}{n \times n-1}$, if $y = \frac{1}{2}$.

To illustrate this Case by an Example in Figures, Let it be required to find the Gain of the Banker when there are 20 Cards remaining in the Stock, and the Ponte's Card twice in it.

$\frac{\frac{1}{2}n+1}{n \times n-1} = \frac{10+1}{20 \times 19} = \frac{11}{380} = \frac{1}{34}$ nearly.

Answer, About the thirty-fourth part of his Stake.

That

That $n \times \frac{n-1}{2}$ is the Sum of all the Chances, may be thus prov'd. 'Tis evident by inſpection of this Scheme, that they are equal to this Series $n-1$, $n-2$, $n-3$, $n-4$, &c. And $\frac{2}{n \times \overline{n-1}} \times \overline{n-1} + \overline{n-2} + \overline{n-3}$, &c. is a Series belonging to the preceding Problem, which expreſſes the Sum of the Probabilities of winning, which belong to the two Gameſters, when the number of all the Cards is n, and the number of red ones two. It therefore expreſſes likewiſe the Sum of the Probabilities of winning which belong to the Ponte and Banker in the preſent Caſe, there being two of the Ponte's Cards in the Stock; but the Sum of theſe Probabilities of winning are equal to Unity, becauſe the Numerators of thoſe Fractions which expreſs their reſpective Probabilities of winning being added together, is equal to the common Denominator, and ſo equal to an Unit, conſequently

$\frac{2}{n \times \overline{n-1}} \times \overline{n-1} + \overline{n-2} + \overline{n-3}$, &c. is equal to an Unit; wherefore putting $s = n-1$, $n-2$, $n-3$, &c. we ſhall have $\frac{2}{n \times \overline{n-1}} \times s = 1$, and therefore $s = n \times \frac{n-1}{2}$.

Solution

Solution of the third CASE.

By the preceding Problem, the number of Chances the Banker has for winning or losing the Stake would be proportional to these Numbers $\overline{n-1} \times \overline{n-2}$, $\overline{n-2} \times \overline{n-3}$, &c. but that on account of the Doublets, the Banker's Chances for winning, as in the preceding Case, are divided into two parts: For instance, $\overline{n-1} \times \overline{n-2}$ is divided into the parts $\overline{n-2} \times \overline{n-3}$, and $2 \times \overline{n-2}$; the former of which is proportional to the Chances that both Banker and Ponte have to win the whole Stake the first pull, because there is the same probability that the Ponte's Card should be drawn the second time and miss'd the first, as drawn the first time and miss'd the second; and the latter part, *viz.* $2 \times \overline{n-2}$ is proportional to the Chances for both first and second Cards being the Ponte's the first pull: and as $2 \times \overline{n-2}$ is easiest found by subtracting $\overline{n-2} \times \overline{n-3}$ from $\overline{n-1} \times \overline{n-2}$; so by subtracting $\overline{n-4} \times \overline{n-5}$, the number of Chances that the Banker and Ponte each have, for winning the whole Stake the next pull from $\overline{n-1} \times \overline{n-4}$, there will remain $2 \times \overline{n-4}$, the number of Chances for the third and fourth Card being the Ponte's, and so for the rest: whence is easily deduc'd the Method of

forming the following Scheme, which ſhews what Chances the Banker has for winning and loſing.

SCHEME.

1	$\begin{Bmatrix} n-2\times n-3 \\ 2\times n-2 \end{Bmatrix}$	Ch. for win.	$\begin{Bmatrix} 1 \\ y \end{Bmatrix}$
2	$n-2\times n-3$	Ch. for loſing	1
3	$\begin{Bmatrix} n-4\times n-5 \\ 2\times n-4 \end{Bmatrix}$	Ch. for win.	$\begin{Bmatrix} 1 \\ y \end{Bmatrix}$
4	$n-4\times n-5$	Ch. for loſing	1
5	$\begin{Bmatrix} n-6\times n-7 \\ 1\times n-6 \end{Bmatrix}$	Ch. for win.	$\begin{Bmatrix} 1 \\ y \end{Bmatrix}$
*	$n-6\times n-7$	Ch. for loſing	1

Hence it appears, that there are no more Chances for winning the Stake than for loſing it; ſo all the Banker's Advantage in this caſe is upon the half Stake, which depends upon drawing of Doublets at one of the Pulls, and all the Chances he has for that may be found thus.

By the above Scheme, $2\times\overline{n-2}$ is the firſt Term of the Series to win y, 4 is the common difference of the reſt of the Terms, and 4 the laſt Term; and ſo the Sum of all the Terms by the known Laws of Arithmetical Progreſſion is $\frac{n\times n-2}{2}$, which divided

vided by the Sum of all the Chances $=$ $\frac{n\times n-1\times n-2}{3}$ by the preceding Problem, gives $\frac{3}{2\times n-1}$ for the probability of winning y; therefore $\frac{3}{2\times n-1}\times y$ is the Banker's Gain, equal $\frac{3}{4\times n-1}$, supposing $y=\frac{1}{2}$.

EXAMPLE.

Query, The Banker's Gain when the Stock consists of 20 Cards, and the Ponte's Card thrice in it?

$\frac{3}{4\times n-1}=\frac{3}{4\times 19}=\frac{3}{76}=\frac{1}{25}$ nearly.

Answer, About the twenty-fifth part of his Stake.

Solution of the fourth CASE.

SCHEME.

$$\begin{array}{r|lll}
1 & \left\{\begin{array}{r} n-2\times n-3\times n-4 \\ 3\times n-2\times n-3 \end{array}\right\} & \text{Ch. for win.} & \left\{\begin{array}{l} 1 \\ y \end{array}\right. \\
2 & n-2\times n-3\times n-4 & \text{Ch. for losing} & 1 \\ \hline
3 & \left\{\begin{array}{r} n-4\times n-5\times n-6 \\ 3\times n-4\times n-5 \end{array}\right\} & \text{Ch. for win.} & \left\{\begin{array}{l} 1 \\ y \end{array}\right. \\
4 & n-4\times n-5\times n-6 & \text{Ch. for losing} & 1 \\ \hline
* & \left\{\begin{array}{l} 2\times 1\times 0 \\ 3\times 2\times 1 \end{array}\right\} & \text{Chance for winning} & \left\{\begin{array}{l} 1 \\ y \end{array}\right. \\
 & 2\times 1\times 0 & \text{Chance for losing} & 1
\end{array}$$

Here alſo the Chances for winning and loſing the Stakes are equal, ſo that the Banker's Gain depends upon y, the half Stake.

And the Chances to win y are

$$\text{Equal to}\left\{\begin{matrix} 3\times2\times1 \\ 3\times4\times3 \\ 3\times6\times5 \\ \&c. \end{matrix}\right\} = 3\times\left\{\begin{matrix} 2\times1 \\ 4\times3 \\ 6\times5 \\ \&c. \end{matrix}\right.$$

And to find the Sum of thoſe Products, *viz.* $2\times1+4\times3+6\times5$, &c. continued to any number of Terms, whoſe Factors are in Arithmetical Progreſſion, we muſt premiſe the following

LEMMA.

Subtract the ſecond Product from the third, and the third Product from the fourth, and call the Remainers firſt Differences; then ſubtract thoſe Differences from each other, and call the Remainer a ſecond Difference. See the following Scheme.

$2\times1=$	2		
$4\times3=$	12	18	8
$6\times5=$	30	26	
$8\times7=$	56		

Now if we call the firſt Product a, the ſecond b, the firſt of the firſt Differences d', the ſecond Difference d''; and if the number

ber of Products which follow the first be called x, the Sum of all the Products will be equal to $a + \overline{x \times b} + \overline{\frac{x}{1} \times \frac{x-1}{2} \times d'} + \overline{\frac{x}{1} \times \frac{x-1}{2} \times \frac{x-2}{3} \times d''}$.

Or

$$2 + \overline{x \times 12} + \overline{\frac{x}{1} \times \frac{x-1}{2} \times 18} + \overline{\frac{x}{1} \times \frac{x-1}{2} \times \frac{x-2}{3} \times 8}.$$

Or

$2 + 5\frac{2}{3}x + 5xx + 1\frac{1}{3}xxx$, which multiplied by 3, the common Multiplier to all the Terms, the Product $6 + 17x + 15x^2 + 4x^3$ will be the Sum of all the Chances for winning of y, whose component Parts, by Sir *Isaac Newton*'s Method of the Invention of Divisors, is readily found to be $2x + 2$, $2x + 4$, and $\frac{4x+3}{4}$; wherefore $\frac{\overline{2x+2} \times \overline{2x+4} \times \overline{4x+3}}{4}$ is the number of Chances for winning of y. Now $x =$ to the number of Terms to be added, bating 1, therefore make $x + 1 = p$, equal to the number of Terms, and then the Chances for winning of y are $\frac{2p \times \overline{2p+2} \times \overline{4p-1}}{4}$. But according to the Scheme it appears, that the number of Terms to be added are equal to $\frac{n-2}{2}$; wherefore writing $\frac{n-2}{2}$ for p, and

and the Chances for winning are equal to $\frac{n\times\overline{n-2}\times\overline{2n-5}}{4}$, which divided by the Sum of all the Chances, *viz.* $\frac{n\times n-1\times n-2\times n-3}{4}$ and the Quotient $\frac{2n-5}{n-1\times n-3}$ is the probability of winning y, and so $\frac{\overline{2n-5\times y}}{n-1\times n-3}$ is the Gain of the Banker, or $\frac{2n-5}{2\times n-1\times n-3}$ supposing $y = \frac{1}{2}$.

EXAMPLE.

Suppose the Stock to consist of 20 Cards, and the Ponte's Card four times in it; *Query*, The Banker's Gain?

$$\frac{2n-5}{2\times n-1\times n-3} = \frac{40-5}{2\times 19\times 17} = \frac{35}{646} = \frac{1}{18}$$ nearly.

Answer, $\frac{35}{346}$ of his Stake accurately, or the eighteenth part of it nearly.

A

A Table of Pharaon, *whereby the ſeveral Advantages of the Banker, in whatever Circumſtances he may happen to be, is ſeen ſufficiently near by inſpection, being calculated from the foregoing Theory.*

N° of Cards in the Stock.	The N° of times the Ponte's Card is contain'd in the Stock.			
	1	2	3	4
52	**	**	**	50
50	**	94	65	48
48	48	90	62	46
46	46	86	60	44
44	44	82	57	42
42	42	78	54	40
40	40	74	52	38
38	38	70	49	36
36	36	66	46	34
34	34	62	44	32
32	32	58	41	30
30	30	54	38	28
28	28	50	36	26
26	26	46	33	24
24	24	42	30	22
22	22	38	28	20
20	20	34	25	18
18	18	30	22	16
16	16	26	20	14
14	14	22	17	12
12	12	18	14	10
10	10	14	12	8
8	8	11	9	6

But

But if an abſolute degree of exactneſs be required, it will be eaſily obtain'd from the Rules and Examples given at the end of each Caſe. However, to make all things as plain as poſſible, I ſhall, to illuſtrate the Uſe of this Table, give an Example or two.

Example 1.

Let it be required to find the Gain of the Banker when there are 30 Cards remaining in the Stock, and the Ponte's Card twice in it.

In the firſt Column ſeek for the Number anſwering to the number of Cards remaining in the Stock: over-againſt it, and under number 2, which is at the head of the Table, you will find 54, which ſhews that the Banker's Gain is the fifty-fourth part of his Stake.

Example 2.

Let it be required to find the Gain of the Banker when there are but 10 Cards remaining in the Stock, and the Ponte's Card thrice in it.

Againſt 10, the number of Cards in the firſt Column, and under number 3, you will find 12, which denotes that the Banker's Gain in this Circumſtance is the twelfth part of his Stake.

Corol-

COROLLARY 1. From the Construction of the Table it appears, that the fewer Cards there are in the Stock, the greater is the Gain of the Banker.

COROL. 2. The least Gain of the Banker under the same circumstance of Cards remaining in the Stock, is, when the Ponte's Card is but twice in it, the next greater when three times, still greater when but once, and the greatest of all when four times.

Of the ACE *of* HEARTS, *or* FAIR CHANCE.

THIS Game is pretty much in vogue, as well as that of *Pharaon*; therefore it may not be improper here to touch a little upon the Advantage or Gain that accrues to the Banker or Taliere at this, as well as of that.

The manner of playing at it is as follows: There is a Table, on which is painted a ſelect number of Cards, generally 31 or 25, or between thoſe Numbers, limited at the fancy of the Perſon who banks the Table; the Player ſtakes upon either of ſuch Cards more or leſs, at his pleaſure. On the Table is fixed an Engine, called a Worm, into which is put an Ivory Ball, which runs round till it drops or falls into a Socket contiguous to one of thoſe Cards; and if it happens to be the Card on which the Player has ſtaked, he ſaves his Stake, and is intitled to 28 or 23 times as much more, according to the number of Cards painted on the Table, and the cuſtom of the Place where

where the Table is kept. From which Deſcription of the Game, the following Problem naturally ariſes.

PROB. III.

The number of Cards upon ſuch a Table, and the number of Stakes the Banker pays in caſe he loſes, being given; to find the Gain of the Banker upon any Sum depoſited as a Stake.

RULE.

FROM the number of Chances or Cards the Table conſiſts of, ſubtract the number of Stakes more by one than the Banker pays when he loſes, and multiply the Remainer by the value of the Stake, and divide the Product by the number of all the Chances or Cards upon the Table, and the Quotient will be the Banker's Gain upon that Stake, and of the ſame Denomination with it; conſequently upon two Cards he will have twice that Advantage, upon three Cards thrice, &c. ſuppoſing the Stakes equal.

EXAM. I.

Suppoſe a Table conſiſts of 31 Cards, and that the Banker pays 28 Stakes when he loſes; *Query*, His Advantage upon a Stake of 100 *l*?

31 The N° of all the Chances.
29 The N° of Stakes paid more by one.
——
2 Difference.

Multiply by 100 the Stake.

———

31)200 (6 *l.*
186
———
14
20
———
31)280 (9 *s.*
279
———
1

Answer, 6 *l.* 9 *s.* $\frac{1}{31}$.

But this Rule is capable of determining the Gain of the Banker when unequal, as well as equal Stakes are set upon two or more different Cards, it being only to be considered as so many distinct Operations.

EXAM. 2.

Suppose *A*, *B*, *C* play, and stake between them 100 *l. viz.* *A* 50, *B* 30, *C* 20; *Query*, The Advantage of the Banker?

1°, 31—29=2 Diff. 2°, 2 Difference.

```
1°, 31—29=2 Diff.          2°, 2 Difference.
          50                    30
       ------                 ------
   31)100(3 l.             31) 60(1 l.
          93                    31
       ------                 ------
           7                    29
          20                    20
       ------                 ------
   31)140(4 s.             31)580(18 s.
         124                    31
       ------                 ------
          16                   270
                               248
                              ----
                                22

3°, 2 Diff.
      20
   31) 40(1 l.
      31
     ---
       9
      20
   31)180(5 s.
     155
     ---
      25
```

			l. s.
Hence the Banker's Gain upon	$\left\{\begin{array}{l} A \\ B \\ C \end{array}\right\}$	is	$\left\{\begin{array}{l} 3,04\frac{16}{31} \\ 1,18\frac{22}{31} \\ 1,05\frac{25}{31} \end{array}\right.$
Total Gain			$6,09\frac{1}{31}$

The ſame as before; whence it follows, that let the Money be ſtak'd how you will, that is, upon as many Cards in what ſhape ſoever you pleaſe, the Banker's Gain will be after the rate of 6 *l.* $9\frac{1}{31}$ *per Cent.* upon all the Money ſtak'd.

But

But for the ſake of a farther Illuſtration, let us ſuppoſe a Perſon to ſtake One Pound Sterling upon each Card, to the number of 31, *viz.* all the Cards upon the Table; in ſuch inſtance, 'tis plain, he's ſure of winning, or to receive 28 *l.* for one of the Cards, and ſave the Stake of that Card which wins; but then 'tis alſo evident upon the whole, that he muſt be a loſer two Pounds, ſince he wins only 28, and loſes 30. Wherefore by the Rule of Proportion it follows, that if in ſtaking 31 Pounds he loſes 2, in ſtaking 100 Pounds he will loſe 6 *l.* 9$\frac{1}{31}$, which conſequently is the Banker's Gain. See the Operation.

l.	*l.*		*l.*	*l.*	*s.*
31 :	2	::	100 :	6.	9$\frac{1}{31}$

```
       100
       ———
31)200 (6 l.
   186
   ———
    14
     20
   ———
31)280 (9 s.
   279
   ———
     1
```

Wherefore

Wherefore the Advantage *per Cent.* is ſo plain, that 'tis needleſs to dwell any longer upon it: However, it may not be amiſs to obſerve, that 'tis an equal Wager that any one Card will win once in 21 times, notwithſtanding the number of all the Chances are 31; which Event is diſcover'd in this or any other Table, by this general

RULE.

From the number of all the Cards upon the Table, deduct one, and multiply the Remainer by ſeven tenths, and the Product is the Anſwer. So in this Caſe,

$$31 - 1 = 30$$

Multiply by $,7$

$$21,0$$

And the Product 21 is the number of Trials requiſite for any one Card to win upon an equality of Chance.

Of

Of LOTTERIES.

IN this place I ſhall conſider the Solution of ſeveral Problems relating to Lotteries, which may be of uſe to prevent ſome miſtakes that People, not vers'd in ſuch Computations, frequently run into. For inſtance, in the preſent Lottery for the Year 1737, where the proportion of Blanks to the Prizes is as 9 to 1, 'tis natural enough to conclude, that 9 Tickets are requiſite for the chance of a Prize; and yet from mathematical Principles 'tis evident, that 7 Tickets are more than ſufficient for that purpoſe, that is, in 7 Tickets it is more likely to have a Prize than not: for this, and all other Caſes of this nature, we ſhall give the Arithmetical Solution of the following Problem.

PROB. IV.

To find how many Tickets muſt be taken, to make it as probable that one or more Prizes may be taken as not.

RULE.

Rule.

Multiply the number of Blanks there are to one Prize by ſeven tenths, and the Product is the Anſwer.

Example.

Query, The number of Tickets requiſite in a Lottery, whereof the number of Blanks is to the number of Prizes as 9 to 1, to make it an equal Chance for one or more Prizes.

$$\begin{array}{r} 9 \\ \underline{,7} \\ \underline{6,3} \end{array}$$

Anſwer, The Product 6,3 ſhows there is more than an equality of Chance in 7 Tickets, but ſomething leſs than an equality in 6.

Example 2.

Query, The number of Tickets requiſite in a Lottery, whereof the number of Blanks is to the number of Prizes as 5 to 1, to make it an equal Chance for one or more Prizes.

$$\begin{array}{r} 5 \\ \underline{,7} \\ \underline{3,5} \end{array}$$

M *Anſwer*,

Answer, The number of Tickets requisite to that effect is between 3 and 4.

PROB. V.

To find how many Tickets must be taken, to make it as probable that two or more Prizes will be taken as not.

RULE.

Multiply 1,678 always by the number of Blanks there are to a Prize, and the Product will be the Answer.

EXAMPLE.

How many Tickets must be had in a Lottery, to make it as probable that two or more Prizes will be taken as not, when there are 9 Blanks to a Prize?

$$\begin{array}{r} 1,678 \\ 9 \\ \hline 15,102 \\ \hline \end{array}$$

Answer, More than 15 Tickets, or rather more than 16, as shall be prov'd farther on, tho' one might undertake upon an equality of Chance to have one at least in 7 Tickets.

The Numbers 0,7 and 1,678 made use of to solve this and the preceding Problem, is the result of determining the Limits of x in these Equations, *viz.* $\overline{a+b}^x = 2b^x$, and

and $\overline{a+b}^x = 2b^x + 2axb^{x-1}$, where a and b repreſent the number of Chances reſpectively for the happening and failing of an Event at any one trial.

'Tis not my deſign to give a Solution to both theſe Equations; but being determin'd to find the Value of x in one of them, I ſhall undertake the laſt, as apprehending it will be moſt acceptable, eſpecially to thoſe who are not very ready in ſolving exponential Equations.

$\overline{a+b}^x = 2b^x + 2axb^{x-1}$, becomes $\frac{\overline{a+b}^x}{b^x} = 2 + 2axb^{-1}$ by dividing both ſides of the Equation by b^x. Make $a:b::1:q$, and the laſt Equation by ſubſtitution becomes $\overline{1+\frac{1}{q}}^x = 2 + \frac{2x}{q}$, in which, if q be ſuppoſed equal to 1, x will be found equal to 3; but if q be ſuppoſed infinite, and $\frac{x}{q} = z$, x will be infinite alſo, and we ſhall have $\overline{1+\frac{1}{q}}^{zq} = 2 \times \overline{1+z}$. Again, the Hyperbolic Log. of $1+\frac{1}{q}$ multiplied by zq is equal to the Hyperbolic Log. of 2 + the Hyperbolic Log. of $1+z$. But the Hyperbolic Log. of $1+\frac{1}{q}$ multiplied by $q = 1$, therefore z = Hyperbolic

 Log.

Log. of 2 + the Hyperbolic Log. of $1+z$. From which last Equation z may be found, for $z - \log. \overline{1+z} = \log. 2 = ,693147 = n$; but the Hyperbolic Log. of $1+z = z - \frac{zz}{2} + \frac{z^3}{3} - \frac{z^4}{4}$, &c. therefore $\frac{z}{2} - \frac{z^3}{3} + \frac{z^4}{4} - \frac{z^5}{5}$, &c. $= n$, and by reversing the Series $z = \sqrt{2n} + \frac{2n}{3} + \sqrt{\frac{n^3}{162}} - \frac{2nn}{135}$, &c. $= 1,6777$, equal to 1,678 nearly.

Hence the Value of x in all Cases will be between $3q$ and $1,678q$; but x converges pretty soon to the last of those Limits, and so the number 1,678, when x is not too small, gives the Answer sufficiently exact; as in the following Example, where the Odds of the Event's happening is greater than in the former.

EXAMPLE 2.

Let it be required to find in how many Throws, one may undertake upon an equality of Chance, to throw three Aces twice, with three Dice?

SOLUTION.

Out of the 216 Chances upon three Dice, there is but 1 Chance for three Aces, and 215 against it; wherefore multiplying the above

above Number 1,678 by 215, and the Product 360,77 ſhows, that 360 Throws, or very near it, are requiſite to produce the required effect.

But when x is ſmall, as in the preceding Example, it needs a correction; for inſtead of 15,102 Tickets, it ſhould be 16,443: which Correction is eaſily had by the Rule of double falſe Poſition. For being aſſur'd that x is found ſomething too little, I therefore aſſume it equal to 16, and ſubſtitute it in the Equation $\overline{1+\frac{1}{q}}^{x} = 2 + \frac{2x}{q}$ and find the left-hand ſide thereof leſs than the right by 0,1589; wherefore I increaſe the value of x four tenths more, *viz.* to 16,4, and ſubſtitute it in the Equation as before, and ſtill find the left-hand ſide too little by 0,0155; then I multiply croſs-ways, and proceed in the reſt of the Operation according to the nature of the Rule, and find $x = 16,443$.

However, we are not deſtitute of a Method whereby the true value of z, and conſequently that of x may be found directly, by the help of an infinite Series, *viz.* $\frac{s}{r} + \frac{s^2}{18r^3} + \frac{1+2r}{162r^5}s^3$, &c. For putting the Hyperbolic Log. of $1+\frac{1}{q} = m$, $mq - \frac{1}{3} = r$, the ſtanding Quantity 1,791759

 $= n$,

$= n$, and $2mq - n = s$. The first Term of the Series, *viz.* $\frac{s}{r}$ subtracted from 2, will give the value of z in this case, true to two places of Decimals, *viz.* 1,83, whence $1,83 \times 9 = 16,47$ is the value of x, or true number of Tickets very near; for three Terms of the Series make $x = 16,44300$. And if x be smaller still, but so as not to have the number of Blanks to a Prize less than 4,1473 (which seldom or never happens in Lotteries) more places of Decimals will turn out true; in short, the above Series will determine the Value of z all Cases, when q is between 4,1473, and any other number how great soever.

Though, as it has been observ'd before, when q is any thing large, $1,678q$ gives x sufficiently near; for two Terms of this Series, in the case of throwing three Aces twice with three Dice, make $z = 1,686$, and consequently $x = 1,686 \times 215 = 362,49$, which is not two throws more than by the former Computation.

Note, The Solution of this Equation, *viz.* $\overline{a+b}^{x} = 2b^x + 2xab^{x-1} + x \times \overline{x-1} \times a^2b^{x-2}$ will give the value of x, in the Case of a triple Event; and the Solution of this, *viz.* $\overline{a+b}^{x} = 2b^x + 2xab^{x-1} + x \times \overline{x-1} \times a^2b^{x-2} + \frac{x}{1} \times \frac{x-1}{1} \times \frac{x-2}{3} a^3b^{x-3}$ will

will give the value of x in the Caſe of a quadruple Event. Here follows a Table of the Limits of x from one to ſix Events incluſive.

The Value of x will always be,

For a	ſingle	Event between	$1q$	and	$0,693q$
	double		$3q$		$1,678q$
	triple		$5q$		$2,675q$
	quadruple		$7q$		$3,672q$
	quintuple		$9q$		$4,670q$
	ſextuple		$11q$		$5,668q$

PROB. VI.

The Number of Tickets a Perſon has in a Lottery being given, to find the Odds againſt him whether they all prove Prizes.

RULE.

To the number of Blanks to a Prize add 1, and make the Sum the Denominator of a Fraction whoſe Numerator is Unity; then multiply this Fraction continually into itſelf as often as the Perſon has Tickets in the Lottery, bating one, and from the laſt Fraction thus produc'd, if Unity be taken from its Denominator, the Remainer will ſhew how many to 1 it is, that they all prove Prizes.

EXAMPLE.

EXAMPLE.

Suppose I have three Tickets in the Lottery of this Year 1737, where there are 9 Blanks to a Prize; how many is it to one but that they are all Prizes?

$$9 + 1 = 10 \text{ the Denominator.}$$

$$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}.$$

Answer, 999 to 1.

N. B. This Rule is only applicable to Lotteries, or in Schemes where there are a great number of Blanks and Prizes.

PROB. VII.

Having the Number of Tickets, and the Number and Amount of all the Prizes undrawn at any time given, to find the Value of a Horse for any number of Days.

RULE.

Multiply the number of Prizes by the Price of an undrawn Ticket, and subtract the Product from the Amount of all the Prizes, and multiply the Remainer by the number of Days the Horse is hired for, and reserve the Product for a Dividend; then multiply the number of undrawn Tickets by the number of Days required to draw them in, and with the Product divide the aforesaid Dividend, and the Quotient will be the Value of the Horse.

EXAMPLE.

EXAMPLE.

Let it be required to find the Value of a Horse for the first Day's drawing in the present Lottery, where there are 70000 Tickets at 10 *l.* each, number of the Prizes 7000, amounting to 226000 *l.* exclusive of the two Prizes for the first and last Numbers drawn, *viz.* 500 and 1000 *l.* and let us suppose that the whole time of Drawing will be 40 Days.

The N° of Prizes	7000
Price of an undrawn Ticket	10
	70000
The N° of undrawn Tickets	70000
The whole time of drawing	40 d.
	2800000

The Amount of all the Prizes, deducting 14 *per Cent.* is 194360 *l.*

```
          194360
           70000
2800000) 124360,000 ( ,044
          11200000      20
          ________     ,880
           12360000     12
           11200000  d.10,560
           ________       4
            1160000  f.2,240
```

Answer, 10 *d.* $\frac{1}{2}$.

 Those

Thoſe that do not chuſe to divide decimally, may, if they pleaſe, multiply the aforeſaid Dividend by 960 before they divide, and the Quotient will be the Anſwer in Farthings; *e. g.*

```
                        124360
                           960
                       -------
                       7461600
                      1119240
                      ---------
        28)00000|1193|85600
        --------
           42     112
                  ---
        4)---      73
          10½      56
                  ---
          ---      17
```

Which 10 *d.* ½ would be the real Value of the Horſe; but as it's on the firſt Day's Drawing, there is a probability of its being the firſt drawn, in conſequence of which the Owner is intitled to a Prize of 500 *l.* this Expectation is worth about three Half-Pence more, *viz.* the 70,000 part of the Value of 500 *l.* when the 14 *per Cent.* is taken off, which being added to the Value before found, makes 1 Shilling, the mathematical Value of a Horſe for the firſt Day's drawing.

EXAMPLE 2.

Admit that 12 Days before the end of the Drawing, there are left in the Wheel of

of Fortune 20544 Tickets, of which 2100 are Prizes, amounting in all to 71400 *l.* *Query*, The Value of a Horſe for two Days, the Price of an undrawn Ticket at that time being worth 11 Pounds.

The Amount of all the Prizes, deducting 14 *per Cent.* is 61404 Pounds.

The Number of Prizes ———	2100
The Price of an undrawn Ticket	11
	2100
	2100
	23100
The Number of undrawn Tickets	20544
Number of Days to draw them in	12
	246528

```
      61404
      23100
      -----
      38304
          2
      -----
246528)76608,000 ( ,310
       739584         20
       ------     s. 6,200
        264960        12
        246528    d. 2,400
        ------         4
        184320     1,600
```

Anſwer, 6 *s.* 2 *d.* $\frac{1}{4}$.

Note, In these Calculations 'tis supposed, (as is customary) that if the Horse proves a Prize during your Jockeyship, that 10 *l.* or an undrawn Ticket be restor'd to the Person who let it.

It is also very plain that this Rule will serve to value the Chance of a Ticket during the whole Lottery, assume the number of Days it will take in drawing what you please; wherefore to render the Operation easy, suppose 1 Day. And altho' I have made a deduction of 14 *per Cent.* upon the Amount of all the Prizes in the two preceding Examples, and at the same time allow the Calculation to be agreeable to the Rules of Art and Science, as being founded on the strictest Demonstration; yet in the present Case, I say, the Deduction is not quite reasonable between Buyer and Seller, and consequently none should be made in finding the value of a Chance for the whole time: For how can it be expected that any one will give up his right in a Ticket if it proves a Prize, and stand to the loss of a Guinea extraordinary if it proves a Blank, *viz.* the Discount upon 7 *l.* 10 *s.* at the rate of 14 *per Cent.* without a valuable consideration, which is that of taking Chances, valued according to the full Amount of the Prizes, and the Price the Tickets bore when they were first purchas'd, *viz.* 10 *l.* each.

This

This Method of proceeding will put both Parties upon an equal footing, than which, I think nothing can be more fair and equitable.

Hence I make the Chance of a Ticket for the whole time of drawing to be worth 2 *l.* 4 *s.* 6 *d.* $\frac{6}{7}$, which, with 2 *d.* $\frac{4}{7}$ for the Expectation of its being either the first or last drawn, makes 2 *l.* 4 *s.* 9 *d.* $\frac{3}{7}$; but if the Chance happens to prove a Prize, 10 *l.* more, or the Price of an undrawn Ticket must be advanc'd.—However, the Market-Price determines what must be given after all; wherefore if a Chance should sell for more than what this Calculation makes, it is not to be wonder'd at, since some consideration ought to be made for the risque that the Dealers in Tickets run in having them sold under par, and for some contingent Expences they are unavoidably at, in furnishing those with Chances and Tickets who are willing to be in Fortune's Way.

But as in all Lotteries Success is precarious, we being kept in suspence till the Event makes known either our good or bad Fortune, so from such state of Uncertainty it follows, that before the Drawing is finish'd a Ticket may be sold for more or less than at present; I shall therefore, before I conclude this Subject, shew how its real worth may be known in any cir-

cumſtance of the Lottery, by which means the value of a Chance may be very accurately determin'd at the ſame time, *e. g.*

Multiply the Number of Blanks remaining in the Lottery at any time of its drawing, by the Price of a Blank, which is always fix'd, and to the Product add the Amount of all the Prizes remaining, the laſt drawn included; this Sum divided by the Number of all the Tickets, *viz.* Blanks and Prizes, will give the value of an undrawn Ticket, which being known, the Value of a Chance for the time, during the remaining part of the Lottery, eaſily flows from the afore-mentioned Rule.

I ſhall conclude this ſmall Tract by making ſome Remarks relating to Hazard and Backgammon; the truth of all which is eaſily deduc'd from the preceding Propoſitions.

Of

Of HAZARD.

1. IF 8 and 6 are Main and Chance, one may lay 155 to 169, or 11 to 12, that either one or the other is thrown off in two Throws.

2. And if 5 and 7, or 9 and 7 are Main and Chance, the probability of their being thrown off in two Throws is alſo as 155 to 169, or as 11 to 12.

3. If 5 and 8, or 9 and 8, or 5 and 6, or 9 and 6 are Main and Chance, the probability of throwing one of them off in two Throws is as 7 to 9 exactly.

4. And if 7 and 4, or 7 and 10 are Main and Chance, the probability of their being thrown off in two Throws is alſo as 7 to 9.

5. If 7 and 8, or 7 and 6 are Main and Chance, one may lay 671 to 625, or 15 to 14 that one of them is thrown off in two Throws, ſo he that lays an even Wager he will throw one of them off in two Throws has the beſt of the Lay.

6.

6. But if 5 and 4, or 5 and 10, or 9 and 4, or 9 and 10 are Main and Chance; he that undertakes to throw either Main or Chance in three Throws has the worſt of the Lay; for it is as 22267 to 24389, or in ſmaller Terms, as 21 to 23 exceeding near; the Ratio of 21 to 23 differing from that of 22267 to 24389 only but by the ten thouſandth part of an Unit.

Note alſo, that 11 and 12 expreſs the Ratio of 155 to 169 the neareſt poſſible in ſuch ſmall Terms, as does 15 to 14 that of 671 to 625, and are eaſily diſcover'd by the Rule exhibited in my Appendix to Dr. *Keil's* Euclid.

7. Suppoſe IV to be a Main, and the Law of the Hazard to be this; That if the Caſter throws either II, III, IV, XI, or XII the firſt Throw, he ſhall loſe the whole Stake, and if he throws V, VI, VII, VIII, IX, or X, either of which, as it may happen, ſhall be deem'd a Chance againſt IV, ſo which ever comes up firſt wins; *Query*, The Hazard of the Caſter to that of the Setter?

Anſwer, The Hazard of the Caſter is to that of the Setter as 457 to 551, or as 5 to 6 very near; wherefore the Gain of the Setter, each Stake being a Guinea, will be $\frac{47}{504}$ equal to 1 *s.* 11 *d.* $\frac{1}{2}$ exactly.

8.

8. And at Hazard, if the Main be 7, and each ſtake a Guinea, the Gain of the Setter is about 3 *d.* $\frac{1}{2}$.

9. If the Main be 6 or 8, the Gain of the Setter is about Six-pence in a Guinea.

10. But if the Main be 5 or 9, the Gain of the Setter is about 3 *d.* $\frac{3}{4}$ in a Guinea; whence it follows, that 5, 7 and 9 are much upon a par to ſet on, and that 6 and 8 are ſomething more advantageous.

11. However, if a Perſon is determin'd to ſet upon the firſt Main that is thrown, his Advantage, ſuppoſing each Stake to be a Guinea, is the $\frac{37}{2016}$ of a Guinea, which when reduc'd will be found equal to 4 *d.* $\frac{1}{2}$, and half a Farthing exactly.

12. Hence the probability of a Main, to the probability of no Main; or, to ſpeak in the gaming Phraſe, a Main or no Main, is as $2016 - 37$ to $2016 + 37$; that is, as 1979 to 2053 accurately, or as 27 to 28 very near; for if one ſtakes 27 Guineas, the other ought not to ſtake quite 2 *d.* $\frac{1}{4}$ more than 28 Guineas, which is a ſmall difference from the truth in ſuch large Stakes as 27 and 28 Guineas.

13. If, with two Dice, one ſhould undertake to throw firſt the two Aces, next the two Duces, next the two Threes, next the two Fours, next the two Fives, and laſtly the two Sixes, the Odds againſt him would be

two thousand, one hundred, and seventy-six Millions, seven hundred, eighty-two thousand, three hundred, and thirty-five to one; and tho' this might possibly happen the first six throws, yet the Odds are so immensely great against it, that it would probably require whole Ages to perform it in: yet notwithstanding all this difficulty in throwing first the two Aces, next the two Duces, *&c.* they may with an equality of Chance be undertaken to be thrown in less than a quarter of an Hour, in the following manner, *viz.* to throw away till the two Aces come up, then till the two Duces, then till the two Threes, and so on till the two Sixes are thrown; but to throw them successively is what, never yet, is rational to suppose, has been done by any one.

14. If any one should undertake to throw a Six or an Ace with two Dice in one throw, he ought to lay 5 to 4, whereas 'tis usual to lay an even Wager only; in which circumstance the Caster has so much the better of the lay, as in the long run to impoverish the best Estates, not to say ruin them. Tho' at first sight it must appear to an Eye not vers'd in these Speculations, a little odd, that the Setter should not have the best of it, since there are but two Sixes, and two Aces for the Caster, and two Fives, two Fours, two Threes, and two Duces,

for

for the Setter. And were the Points of both Dice all made upon a regular Solid or Body of 12 equal Faces, ſuch as the Dodecaedron, the Caſter would undoubtedly have the advantage; for then he would have two to one of the lay, in as much as he would have 8 Chances for winning, and but 4 for loſing: But as there are two Dice, it muſt be conſidered as the happening of two Events, independent of each other, which makes the Odds, as I ſaid before, juſt 5 to 4.

COROLLARY.

Hence it follows, that at Backgammon if two Points are open, 'tis 5 to 4 but that a Perſon enters the firſt throw; and as this Thought naturally leads me to give a Solution of the reſt of the Hazards, it may perhaps be acceptable if I ſhew the Odds of entring when other Points of the Table are open, and therefore ſhall give the following Scheme for that purpoſe.

SCHEME.

Points open	A Perſon may lay			
1	11	to	25	that one enters.
2	5	to	4	that one enters.
3	3	to	1	that one enters.
4	8	to	1	that one enters.
5	35	to	1	that one enters.

And

And I don't doubt, but the Knowledge of theſe Odds may enable one to play the Game in other reſpects with great advantage; tho' for my part I own with regard to practice, that I have but very little ſkill in this, or any other Game whatſoever.

FINIS.

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